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NUMERICAL ALGORITHMS FOR OSCILLATION VECTORS OF SECOND ORDER DIFFERENTIAL EQUATIONS INCLUDING THE EULER-LAGRANGE EQUATION FOR SYMMETRIC TRIDIAGONAL MATRICES

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We give numerical algorithms for second order differential equations. More specifically we consider the problem of numerically determining oscillation points and vectors for numerical solutions of the equation $(r(t)x'(t))'+p(t)x(t)=0$ and focal points and vectors for the quadratic form $J(x)=\int_a^b(rx'^2-px^2)dt$.

As a biproduct of our work we obtain some new theoretical and numerical results for symmetric tridiagonal matrices. Many of our results may be extended to eigenvalue and eigenvector problems, integral and partial differential equations, and higher order problems described in § 4. Many of our matrix results may be extended to more general banded symmetric matrices.

In a broad sense this work is a numerical application of an approximation theory of quadratic forms on Hilbert spaces given by the author. Our ideas are based on generalizations of a basic idea of Hestenes; namely on the consideration of the "negative signature of quadratic forms [5]." Our ideas are similar to finding roots of polynomials by looking at sign changes as opposed to the more difficult problem of solving equations.

For convenience of presentation we now describe the basic procedure: (i) Let $(r(t)x'(t))' + p(t)x(t) = 0$ be the differential equation which is the Euler-Lagrange equation of the quadratic form $J(x)=\int_a^b(rx'^2 - px^2)dt$. Let $x_0(t)$ be "a" solution of the differential equation satisfying $x_0(a) = 0$. (ii) Approximate the vectors $x(t)$ on $[a, b]$ by Spline functions of degree 1 (order 2) so that the approximating finite dimensional quadratic form is $J(x_\mu; \mu) = x_\mu^T D_\mu x_\mu$; where x_μ is a "piecewise linear function," D_μ is a symmetric tridiagonal matrix, and μ is a parameter denoting the distance between "knot points." (iii) Obtain the "Euler-Lagrange equations" for D_μ; call the solution $c(\mu)$. (iv) Show that $c(\mu)$ converges to $x_0(t)$ as $\mu\to 0$ in the strong L^2 derivative norm sense. We remark that we have previously shown that the negative signature (negative eigenvalues) of the matrix D_μ "agrees" with the negative signature of the quadratic