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ON COUNTABLE PRODUCTS AND ALGEBRAIC CONVEXIFICATIONS OF PROBABILISTIC METRIC SPACES

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Two different ways of defining a probabilistic metric on the countable product of a family of probabilistic metric spaces are studied and compared. The algebraic convexification of probabilistic metric spaces is also investigated.

0. Introduction. Finite products of probabilistic metric (PM) spaces have been studied previously by R. Egbert [1], R. Tardiff [10], A. Xavier [13] and V. Istratescu and I. Vaduva [2]. In this paper we turn to the study of countable products.

If $\{(S_i, \mathscr{F}^i, \tau_i) \mid i \in N\}$ is a family of PM spaces and if we form the generalized metric space $(\prod_{i=1}^{\infty} S_i, \prod_{i=1}^{\infty} \Delta^+, \prod_{i=1}^{\infty} \tau_i)$ in the sense of E. Trillas [11, 12], then the problem is to choose the most satisfactory assignment of a probability distribution function in Δ^+ to each member of the family $(\mathscr{F}^i)$, i.e., to each sequence $(F_i) \in \prod_{i=1}^{\infty} \Delta^+$. Two natural assignments are considered:

(a) The series $\sum_{i=1}^{\infty} (1/2^i) F_i$ as the weak limit of the pointwise nondecreasing sequence $\{\sum_{i=1}^{n} (1/2^i) F_i \mid n \in N\}$ in Δ^+.

(b) The product $\tau_{i=1}^{\infty} F_i$ as the weak limit of the pointwise nonincreasing sequence $\{\tau(F_1, \cdots, F_n) \mid n \in N\}$ in Δ^+, where τ is an arbitrary triangle function.

In case (a) we speak of *Σ-products* and in case (b) of *τ-products*.

In addition we also consider the question of the algebraic convexification of a PM space, which involves the embedding of the given space a in convex subspace of a suitably defined countable product.

Throughout the paper we assume that the reader is familiar with the basic definitions and concepts of the theory of PM spaces as given, e.g., in [8] or [10].

1. On Σ-products. We begin with the following:

DEFINITION 1.1. Let $\{(S_i, \mathscr{F}^i, \tau_i) \mid i \in N\}$ be a countable family of PM spaces. The Σ-product of this family is the space $(\prod_{i=1}^{\infty} S_i, \mathscr{F}^{\Sigma})$, where $\mathscr{F}^{\Sigma}\colon \prod_{i=1}^{\infty} S_i \times \prod_{i=1}^{\infty} S_i \to \Delta^+$, is the mapping given by $\mathscr{F}^{\Sigma}((p_i), (q_i)) = \sum_{i=1}^{\infty} (1/2^i) \mathscr{F}^i(p_i, q_i)$, for any sequences (p_i) and (q_i) in $\prod_{i=1}^{\infty} S_i$.

In this section we will use the abbreviations: $S = \prod_{i=1}^{\infty} S_i$, $F = \mathscr{F}^{\Sigma}$, $F_{\overline{pq}} = \mathscr{F}^{\Sigma}((p_i), (q_i))$ and $F_{p_i q_i} = \mathscr{F}^i(p_i, q_i)$.