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REPRESENTATIONS OF THE MAUTNER GROUP, I

LARRY BAGGETT

The five-dimensional connected Lie group known as the "Mautner group" is not of type I. In fact all of its irreducible unitary representations are not yet known. We present here a discussion of the known representations and produce a five-parameter family of new representations.

I. Introduction. By the *Mautner group* we shall mean the semidirect product $G = C^2R$ of two-dimensional complex space C^2 with the real line R in which multiplication is given by $((z, w, t)(z', w', t')) = (z + e^{it}z', w + e^{2\pi it}w', t + t')$. This group is distinguished by being the "Smallest" connected Lie group whose unitary dual, space of equivalence classes of irreducible unitary representations, remains unknown. In particular, Mackey's theory for semi-direct products is insufficient in this case, the Mautner group not being a "Regular" semi-direct product. Our goal has been to develop new techniques for constructing irreducible representations of G which would supplement Mackey's method and hopefully provide a realization of all the irreducible unitary representations of the Mautner group together with a reasonable criterion for their mutual equivalence. This group is not of type I, (indeed Mautner himself discovered it as the first example of a connected Lie group which was not of type I), so that its unitary dual is not "Smooth." We will then not be able to catalogue the entire dual by any "Nice" space of parameters, but we can hope that an adequate description of the dual, if less precise, does exist. For example, in this first paper we shall produce a series $[U^{(r,s,\lambda,\mu,d)}]$ of irreducible representations of G, with r and s positive real numbers, λ and μ real numbers, and d an integer, which satisfies:

THEOREM 1.1. *$U^{(r,s,\lambda,\mu,d)}$ is equivalent to $U^{(r',s',\lambda',\mu',d')}$ if and only if $r = r'$, $s = s'$, $d = d'$, and $\lambda + d\mu = \lambda' + d'\mu' + p + 2\pi q$, for some integers p and q. For d unequal to zero, the representations $[U^{(r,s,\lambda,\mu,d)}]$ are "New" representations of G.*

We mention here that the theory of representations induced from virtual subgroups of G, invented by Mackey and developed by Ramsay in [5], classifies completely, in a sense, the entire unitary dual. Really it only replaces the problem of constructing the irreducible unitary representations of G by the two problems: (a) finding all the virtual subgroups of G, (very difficult and not yet solved even in this simplest case), and (b) finding all the irreducible representations of these