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PROJECTIVE MODULES OVER SUBRINGS OF $k[X, Y]$ GENERATED BY MONOMIALS

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In this paper we study finitely generated projective modules over affine subrings A of $k[X, Y]$ generated by monomials. If A is normal, then all finitely generated projective A-modules are free. If A is not normal, we show that finitely generated projective A-modules stably have the form free $\oplus$ rank one

1. Introduction. In this paper we study projective modules over subrings A of $k[X, Y]$ generated by monomials. We study conditions on A so that all finitely generated projective A-modules have the form free $\oplus$ rank one. In §IV we use Seshadri's localization technique to show that all finitely generated projective A-modules are free when A is an affine normal subring of $k[X, Y]$ generated by monomials. If we drop the assumption that A is normal it need not be true that all finitely generated projective A-modules are free. However, in §V we show that finitely generated projective A-modules stably have the form free $\oplus$ rank one. We also give sufficient conditions on k for finitely generated projective A-modules to have the form free $\oplus$ rank one. These results do not generalize to arbitrary subrings of $k[X, Y]$.

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2. Preliminaries. All rings A will be commutative with 1. Spec(A) is the set of all prime ideals of A and max(A) is the subset of spec(A) consisting of maximal ideals. We give spec(A) the Zariski topology. If X is a topological space, the combinatorial dimension of X will be denoted by $\dim X$. If A is a ring, the group of units of A is A^*. $\mathrm{SL}(n, A)$ is the group of $n \times n$ matrices over A with determinant 1, and $E(n, A)$ is the subgroup of $\mathrm{SL}(n, A)$ generated by elementary matrices. The Krull dimension of A will be denoted by $\dim A$. k will always be a field. Let P be a finitely generated projective A-module and $Q \in \operatorname{spec}(A)$. We define $\operatorname{rank}_Q P$ to be $\dim_{A_Q/QA_Q} P_Q/QP_Q$. If $\operatorname{rank}_Q P$ is constant, we will denote it by $\operatorname{rank} P$. Our K-theory notation will follow Bass [4].

$\tilde{K}_0(A)$ is the subgroup of $K_0(A)$ generated by $[A^{\operatorname{rank} P}] - [P]$ for finitely generated projective A-modules P, and Pic(A) is the group of isomorphism classes of finitely generated projective A-modules of rank