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CONNECTIVITY PROPERTIES OF METRIC SPACES

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We discuss various connectivity properties of a metric space, and investigate how far their equivalence carries over from the classical to the constructive setting. In passing, we obtain interesting relations between connectivity and convexity for subsets of $\boldsymbol{R}$, and a result on preservation of connectivity by continuous mappings.

1. The primary object of this note is a constructive examination of the relationship between several, classically equivalent connectivity properties of a metric space (E, d). In order to make sense of the statements of these properties, we recall that a subset A of E is *located* (in E) if

$$\operatorname{dist}(x, A) \equiv \inf\{d(x, a) : a \in A\}$$

is computable for each x in E; in which case the *metric complement* of A in E is defined to be

$$E - A \equiv \{x \in E : \operatorname{dist}(x, A) > 0\}\ .$$

Note that a located set A is nonvoid, in the sense that we can construct at least one of its elements. For further properties of located sets, and general background material in constructive analysis, we refer the reader to [1] and [2].

In [3], we introduced the following types of connectivity of a metric space:

C-connectivity: if A is a closed, located subset of E with nonvoid metric complement, then there exists a point ξ in $A \cap (E - A)^-$;

0-connectivity: if A is an open, located subset of E with nonvoid metric complement, then there exists ξ in $\bar{A}$ such that $d(\xi, x) > 0$ for each x in A;

Connectivity: if A is an open, closed and located subset of E, then $A = E$.

We then showed that

$$C\text{-connectivity} \Longrightarrow 0\text{-connectivity} \Longrightarrow \text{connectivity}\ .$$

In this section, we shall show that these implications cannot be reversed