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SHIFTS ON INDEFINITE INNER PRODUCT SPACES

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We use the concept of a wandering subspace to study isometries on spaces with an inner product that is not assumed to be positive definite. The theory in many respects parallels the Hilbert space theory, but there are significant differences that are emphasized here. Examples are given which illustrate the complications that can arise when the inner product is indefinite.

The first few sections of this paper are devoted to the study of indefinite inner product spaces with admissible topologies, and the continuous operators on these spaces. The rest of the paper concentrates on isometric operators, their wandering subspaces, and the Fourier representations of shifts.

1. Introduction. In the paper [5], Halmos studies shifts on Hilbert spaces by using wandering subspaces. We apply this technique here, where we consider isometries on indefinite inner product spaces. These operators have been studied in the past, principally by Iohvidov ([6] and [7]), on spaces where the indefinite inner product is derived from a Hilbert space inner product (a J- or G-inner product). The results obtained for isometries here, however, apply in the more general situation of an inner product space with an admissible topology.

The theory of shifts on Hilbert space was used by Sz.-Nagy and Foiaş [11] in the study of the geometry of spaces of minimal unitary dilations of contractions. A noncontraction possesses a minimal unitary dilation on a space with an indefinite inner product [2], and it was the study of the geometry of these dilation spaces (originating in the papers of Davis [2] and Davis and Foiaş [3]) that motivated the present work.

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2. Inner product spaces. An *inner product space* is a complex vector space $\mathscr{H}$ with an inner product $[\cdot, \cdot]$ satisfying

$$[\alpha_1 h_1 + \alpha_2 h_2, k] = \alpha_1[h_1, k] + \alpha_2[h_2, k]$$

and

$$[k, h] = \overline{[h, k]}$$