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PARTIAL ORDERINGS FOR INTEGRAL REPRESENTATIONS ON CONVEX SETS WITH THE RADON-NIKODYM PROPERTY

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The separable extremal ordering and the dilation ordering (as well as equivalent reformulations) have been extensively used in the study of integral representations on closed bounded convex sets with the Radon-Nikodym Property. The relationship between these orderings is clarified and other orderings are introduced.

Considerable attention has recently been focused on closed bounded convex sets which have the Radon-Nikodym Property. Such sets exhibit behavior reminiscent in many instances of weakly compact convex sets, and in particular, a theory of integral representations has emerged for these sets which closely parallels that developed by Choquet, Bishop, DeLeeuw and others for compact convex sets. As with the compact case, in all but the most elementary situations (i.e., separable closed bounded convex sets, which correspond in simplicity to metrizable compact convex sets) the techniques center about the use of partial orderings as a means of determining how 'close' to the 'boundary' of the convex set the various measures under consideration live. The exact relationship between two of these orderings, the dilation ordering $\prec_d$ and the separable extremal ordering of Mankiewicz $\prec_m$, is the main subject of this paper.

Each of [17], [4], and [1] is recommended for a comprehensive review of the compact case. Several partial orderings have been used in the noncompact case: Edgar [7] studied the dilation ordering in connection with a general existence theorem, while Bourgin and Edgar [3] used $\prec_d$ and another partial ordering introduced by Edgar in [7], here denoted by $\prec_e$, to prove uniqueness. Independently, St. Raymond [20] employed the Choquet ordering, $\prec_c$, to study the uniqueness question in the separable case. Then Mankiewicz [13] introduced the separable extremal order, and provided a significantly easier proof of the existence of maximal integral representations than had previously been available. Some relationships between these orderings were obtained in [7], [3], [20], [13], and [8], and those needed in this paper are listed in Theorem 1.3.

Section 1 contains background material. The main results, formulated in a variety of ways, are contained in Theorems 2.1, 2.4, 2.5, and Corollary 2.3. Various other partial orderings are suggested by Theorem 2.1 and these are considered in the latter part of §2.