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NECESSARY CONDITIONS AND SUFFICIENT CONDITIONS FOR DISFOCALITY AND DISCONJUGACY OF A DIFFERENTIAL EQUATION

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A necessary and sufficient criterion for the $(k, n-k)$-disfocality of the equation $y^{(n)} + p(x)y = 0$, $a < x < b$, is proved. This criterion is used to establish explicit necessary conditions for disfocality in terms of lower order equations and integral inequalities. The same criterion is used to obtain sufficient conditions, expressed by inequalities of similar type.

1. Introduction.

An nth order linear differential equation

$$y^{(n)} + p_i(x)y^{(n-1)} + \cdots + p_n(x)y = 0\ ,$$

whose coefficients are continuous on an interval I, is said to be *disconjugate* on I if none of its nontrivial solutions has n zeros in I (including multiplicities). If an equation is not disconjugate on $[a, b]$, the conjugate point of a is defined as the infimum of the values $t, t > a$, such that the equation is not disconjuate on $[a, t]$. If the conjugate point of a exists, it is denoted by $\eta(a)$. There exists a solution associated with the interval $[a, \eta(a)]$, which has a zero of multiplicity k at $x = a$ and a zero of multiplicity at least $n-k$ at $x = \eta(a)$ for certain $k, 1 \leqq k \leqq n-1$, and which does not vanish in $(a, \eta(a))$.

The subject of this paper is the disconjugacy of the equation

$$y^{(n)} + p(x)y = 0\ , \tag{1}$$

where $p(x)$ is of constant sign. For (1) we have further information about the solution associated with $[a, \eta(a)]$. It has a zero exactly of multiplicity k at $x = a$ and a zero exactly of multiplicity $n-k$ at $x = \eta(n)$. Moreover, $n-k$ is odd if $p(x) \geqq 0$ and $n-k$ is even if $p(x) \leqq 0$ [16].

The distribution of the zeros of the solution associated with $[a, \eta(a)]$ suggests the following definition: Equation (1) is said to be $(k, n-k)$-*disconjugate* on an interval I if for every pair of points $a, b \in I$, $a < b$, there does not exist a nontrivial solution of (1) which satisfies

$$\begin{aligned} y^{(i)}(a) &= 0\ , \quad i = 0, \cdots, k-1\ , \\ y^{(j)}(b) &= 0\ , \quad j = 0, \cdots, n-k-1\ . \end{aligned} \tag{2}$$