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A TREE-LIKE TSIRELSON SPACE

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An example is given of a reflexive Banach space X such that $(X \oplus X \oplus \cdots \oplus X)_{l_1^n}$, $n = 1, 2, \cdots$, are uniformly isomorphic to X. Some related examples are also given.

1. Introduction. In [4] Lindenstrauss observed that a Banach space X such that $(X \oplus X \oplus \cdots \oplus X)_{l_1^n}$ is isometric to a subspace of X for every n must contain an isometric copy of l_1. This gives a very simple proof to the fact that there exists no separable reflexive Banach space which is isometrically universal for all the separable reflexive Banach spaces. Lindenstrauss asked whether the isomorphic version of this result is true; i.e., does the fact that X contains uniformly isomorphic images of $(X \oplus X \oplus \cdots \oplus X)_{l_1^n}$, $n = 1, 2, \cdots$, imply that X contains l_1 isomorphically? An affirmative answer would give an alternative proof to the nonexistence of an isomorphically universal space in the family of all separable reflexive spaces as well as in the family of all spaces with a separable dual. (The nonexistence of these spaces was proved by W. Szlenk [8] by a completely different method.) Unfortunately the answer to Lindenstrauss' question is negative in a very strong sense.

THEOREM. *Let $1 \leqq p \leqq \infty$ and $\lambda > 1$. There exists a Banach space X with a 1-unconditional basis $\{e_i\}_{i=1}^\infty$ with the following properties:*

(a) *X is reflexive.*

(b) *X does not contain a subspace isomorphic to l_p (c_0 in the case $p = \infty$).*

For every $n = 1, 2, \cdots$ there exist n disjoint subsequences of the natural numbers $N_1, N_2, \cdots, N_n$ such that

(c) *$\{e_i\}_{i \in N_j}$; is isometrically equivalent to $\{e_i\}_{i=1}^\infty$, and*

(d) *If $x_j \in [e_i]_{i \in N_j}$; $j = 1, 2, \cdots, n$ then*

$$\lambda^{-1}\Big(\sum_{j=1}^n \|x_j\|^p\Big)^{1/p} \leqq \Big\|\sum_{j=1}^n x_j\Big\| \leqq \lambda\Big(\sum_{j=1}^n \|x_j\|^p\Big)^{1/p}$$
$$\Big(\lambda^{-1} \max_{1 \leqq j \leqq n} \|x_j\| \leqq \Big\|\sum_{j=1}^n x_j\Big\| \leqq \lambda \max_{1 \leqq j \leqq n} \|x_j\| \ \textit{if}\ p = \infty\Big).$$

(e) *There exists a $K < \infty$ such that X is K-isomorphic to $(X \oplus X \oplus \cdots \oplus X)_{l_p^n}$ for every n.*

The construction uses ideas from [9] and [1] as well as the basic