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# OPERATORS SATISFYING A $G_1$ CONDITION

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**An operator $T$ on a Hilbert space is said to be $G_1$ if $\|(T-z)^{-1}\|=1/\mathrm{dist}(z, \sigma(T))$ for $z \notin \sigma(T)$ and completely $G_1$ if, in addition, $T$ has no normal part. Certain results are obtained concerning the spectra of completely $G_1$ operators and of their real parts. It is shown in particular that there exist completely $G_1$ operators having spectra of zero Hausdorff dimension. Some sparseness conditions on the spectrum are given which assure that a $G_1$ operator has a normal part.**

**1. Introduction.** All operators considered in this paper will be bounded (linear) on a Hilbert space $\mathfrak{H}$ of elements $x$. For any such operator $T$ it is well-known (and due to Wintner [26]) that

$$\|(T-z)^{-1}\| \geqq 1/\mathrm{dist}(z, \sigma(T))$$

for $z \notin \sigma(T)$ and $\|(T-z)^{-1}\| \leqq 1/\mathrm{dist}(z, W^{-}(T))$ for $z \notin W^{-}(T)$, where $\sigma(T)$ denotes the spectrum of $T$ and $W^{-}(T)$ denotes the (convex) closure of the numerical range $W(T) = \{(Tx, x): \|x\| = 1\}$. An operator $T$ is said to be $G_1$ (or to satisfy a $G_1$ condition, or to be of class $G_1$) if

$$\|(T-z)^{-1}\| = 1/\mathrm{dist}(z, \sigma(T)) \quad \text{for} \quad z \notin \sigma(T)\,. \tag{1.1}$$

For instance, (1.1) holds for operators $T$ which are normal ($T^*T - TT^* = 0$), more generally, for those which are subnormal ($T$ has a normal extension on a larger Hilbert space), and still more generally, for hyponormal operators ($T^*T - TT^* \geqq 0$). The inclusions indicated here,

$$\text{normals} \subset \text{subnormals} \subset \text{hyponormals} \subset (G_1)\,, \tag{1.2}$$

are all proper and, needless to say, the simple stratification (1.2) can be interstitially (and endlessly) refined. In this connection, see the brief survey in Putnam [16].

An operator $T$ will be called completely $G_1$ if $T$ is $G_1$ and if, in addition, $T$ has no normal part, that is, $T$ has no reducing subspace on which it is normal. Similarly, one has corresponding definitions of completely subnormal or completely hyponormal operators. It is well-known that every compact set of the plane is the spectrum of some normal operator. Moreover, necessary and sufficient conditions are known in order that a compact set be the spectrum of a completely subnormal operator (Clancey and Putnam [4]) or of a completely