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ON THE REDUCTION OF CERTAIN DEGENERATE PRINCIPAL SERIES REPRESENTATIONS OF SP (n, C)

THOMAS A. FARMER

This paper has its origins in the problem of proving irreducibility or reducibility for principal series representations of certain noncompact, complex, semi-simple groups by Fourier-analytic methods; for example, the abelian methods of Gelfand-Naimark for Sl (n, C), and the non commutative (nilpotent) methods of K. Gross for Sp (n, C). As is well-known, principal series representations are induced from unitary characters of a parabolic subgroup, the series being termed "nondegenerate" if the parabolic is minimal (i.e., the Borel subgroup) and otherwise "degenerate". Here we consider degenerate principal series for Sp (n, C) corresponding to maximal parabolic subgroups (more general than the situation studied by Gross) and reduce them with respect to the "opposite" parabolic. Let n_1 denote the complex dimension of the isotropic subspace corresponding to the maximal parabolic, let $0 < n_1 < n$, and $n_0 = n - n_1$. The resulting reduction is described in terms of the natural representation of the complex orthogonal group $O(n_1, C)$ acting on the space $L^2(C^{n_1 \times n_0})$ and the tensor product of n_1 copies of the oscillator representation of Sp (n_0, C). In the terminology introduced by R. Howe, this harmonic analysis reduces to the theory of a "dual reductive pair", and any further resolution of the question of irreducibility by these methods will depend upon the study of the oscillator representations for such a dual reductive pair.

We now describe our work in more detail. As a presentation of the complex symplectic group, take

$$\sum_n = \{g \in C^{2n \times 2n}: gM_n g' = M_n\} ,$$

where $M_n = \begin{bmatrix} 0 & -I_n \\ I_n & 0 \end{bmatrix}$, I_n is the $n \times n$ identity matrix, and g' denotes the transpose of g. Specify a complete set of conjugacy class representatives of the maximal parabolic subgroups H in Σ_n (c.f., [9], §8) by defining $H = Z'SA$, where the subgroups Z, S, and A are given below. Let the isotropic subspace of C^{2n} corresponding to H have dimension n_1, with $0 < n_1 \leqq n$ and $n_0 = n - n_1$. Then the blocking scheme used in defining Z, S, and A has diagonal blocks of dimensions $n_1 \times n_1$, $n_0 \times n_0$, $n_1 \times n_1$, and $n_0 \times n_0$ from upper left to lower right.