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SOME GENERALIZATIONS OF CARLITZ'S THEOREM

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Recently, L. Carlitz extended certain known generating functions for Laguerre and Jacobi polynomials to the forms:

$$\sum_{n=0}^{\infty} c_n^{(\alpha+\lambda n)} \frac{t^n}{n!} \quad \text{and} \quad \sum_{n=0}^{\infty} d_n^{(\alpha+\lambda n, \beta+\mu n)} \frac{t^n}{n!},$$

respectively, where $c_n^{(\alpha)}$ and $d_n^{(\alpha,\beta)}$ are general one- and two-parameter coefficients. In the present paper some generalizations of Carlitz's results of this kind are derived, and a number of interesting applications of the main theorem are given.

1. Introduction and the main results. Motivated by his generating function [2, p. 826, Eq. (8)]

$$\sum_{n=0}^{\infty} L_n^{(\alpha+\lambda n)}(x) t^n = \frac{(1+v)^{\alpha+1}}{1-\lambda v} \exp(-xv), \tag{1.1}$$

where α, λ are arbitrary complex numbers and v is a function of t defined by

$$v = t(1+v)^{\lambda+1}, \qquad v(0) = 0, \tag{1.2}$$

and by its subsequent generalization due to Srivastava and Singhal [9, p. 749, Eq. (8)]

$$\begin{aligned} &\sum_{n=0}^{\infty} P_n^{(\alpha+\lambda n, \beta+\mu n)}(x) t^n \\ &\quad = (1+\xi)^{\alpha+1}(1+\eta)^{\beta+1}[1-\lambda\xi-\mu\eta-(1+\lambda+\mu)\xi\eta]^{-1}, \end{aligned} \tag{1.3}$$

where ξ and η satisfy

$$(x+1)^{-1}\xi = (x-1)^{-1}\eta = \frac{1}{2}t(1+\xi)^{\lambda+1}(1+\eta)^{\mu+1}, \tag{1.4}$$

Carlitz [3] has recently derived generating functions for certain general one- and two-parameter coefficients [op. cit., p. 521, Theorem 1 and Eq. (2.10)]. Our proposed generalizations of Carlitz's main results in [3] are contained in the following

THEOREM. *Let $A(z)$, $B(z)$ and $z^{-1}C(z)$ be arbitrary functions which are analytic in the neighborhood of the origin, and assume that*

$$A(0) = B(0) = C'(0) = 1. \tag{1.5}$$