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THE PROJECTIVITY OF EXT (T, A) AS A MODULE OVER $E(T)$

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Let A and T be abelian groups. Then $\operatorname{Ext}(T, A)$ can be considered as a right module over $E(T)$, the ring of endomorphisms of T. In this paper necessary and sufficient conditions are developed for $\operatorname{Ext}(T, A)$ to be $E(T)$-projective whenever T is reduced torsion and A is reduced.

In this paper A and T will be abelian groups and $\operatorname{Ext}(T, A)$ will be considered as a right $E(T)$-module. (See [5].) We consider the question of when $\operatorname{Ext}(T, A)$ is a projective $E(T)$-module. Theorems 1 and 2 provide necessary and sufficient conditions for $\operatorname{Ext}(T, A)$ to be $E(T)$-projective whenever T is reduced torsion and A is reduced. It is interesting to note (Theorem 3) that if B is any reduced group, a necessary condition for $\operatorname{Ext}(B, A)$ to be $E(B)$-projective is that $\operatorname{Ext}(B, A) \simeq \operatorname{Ext}(T(B), A)$. Hence if $\operatorname{Ext}(B, A)$ is $E(B)$-projective, $\operatorname{Ext}(B, A) \simeq \operatorname{Ext}(T(B), A)$ and $\operatorname{Ext}(T(B), A)$ may be considered as an $E(T(B))$-module, where $T(B)$ is, of course, reduced torsion.

We shall employ the following notations and conventions: The word "group" will always mean "abelian group." We reserve the letter T for a torsion group, and in this case, T_p will be the p-primary component of T. For an arbitrary group A, $T_p(A)$ is the p-primary component of the torsion part of A. For a ring R and a left R-module M, $r_R(M)$ will refer to the rank of M as defined in [4], $hd_R(M)$ and $id_R(M)$ will refer, respectively, to the homological and injective dimensions of M as defined in [6]. An isomorphism of R-modules M and N will be denoted by: $M \overset{R}{\simeq} N$. Other notations will follow [2]. Importantly, whenever we speak of $\operatorname{Ext}(T, A)$ as a right $E(T)$-module we may assume without loss of generality that A is reduced as a group. Finally, if $A \overset{Z}{\simeq} (v) \oplus A'$, and if $a \in A$, we will write, conveniently, when defining an endomorphism α of A: $\alpha(v) = a$, $\alpha = 0$ otherwise. We mean, more precisely, that: $\alpha(v) = a$, $\alpha|_{A'} = 0$. We now state our main theorems:

THEOREM 1. *Let T be a reduced p-primary group and let A be a reduced group. Then* $\operatorname{Ext}(T, A)$ *is a projective right* $E(T)$-*module if and only if either* $\operatorname{Ext}(T, A) = 0$, *or all of the following conditions hold:*

(i) *T is bounded, with minimal annihilator p^k, say.*