

THE SESSILE LIQUID DROP I. SYMMETRIC CASE

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Quantitative estimates are derived, describing the size and shape of a symmetric (idealized) liquid drop, resting in gravitational equilibrium on a plane surface Π . The free surface interface is determined by the conditions that its mean curvature be a given (increasing) linear function of distance from Π , that it enclose with Π a prescribed volume V , and that the angle formed with Π be a prescribed constant γ . The estimates apply to drops of all sizes, and some are asymptotically exact in the limiting cases of large or small wetted area on Π . It is shown that a number of qualitative features of behavior are determined by the ratio $V/\sin \gamma/2$. This ratio is in turn related to a ratio that appears in the study of the circular capillary tube, thus indicating a reciprocity between the two problems, which becomes exact in both limiting cases.

As corollaries of the method, the uniqueness of the symmetric solution is proved, and a new proof of existence is given.

The results are compared with calculations and with measurements in some particular cases.

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1. Introduction. The existence of a stationary (sessile) liquid drop of prescribed volume V and minimizing potential energy, rest-