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REGULARITY OF CAPILLARY SURFACES OVER DOMAINS WITH CORNERS

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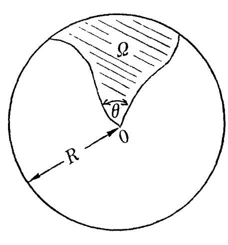
Using the usual mathematical model (capillary surface equation with contact angle boundary condition) we discuss regularity of the equilibrium free surface of a fluid in a cylindrical container in case the container cross-section has corners.

It is shown that good regularity holds at a corner if the "corner angle" θ satisfies $0<\theta<\pi$ and $\theta+2\beta>\pi$, where $0<\beta\leq \pi/2$ is the contact angle between the fluid surface and the container wall.

It is known that no regularity holds in case $\theta+2\beta<\pi$, hence only the borderline case $\theta+2\beta=\pi$ remains open.

We here want to examine the regularity of solutions of capillary surface type equations (subject to contact angle boundary conditions) on domain $\Omega \subset \boldsymbol{R}^2$ in a neighbourhood of a point of $\partial\Omega$ where there is a corner.

To be specific let Ω (as depicted in the diagram) be a region contained in $D_R = \{x \in \boldsymbol{R}^2 \colon |x| < R\}$ $(R > 0$ given) such that $\partial\Omega$ consists of a circular segment of ∂D_R together with two compact Jordan arcs γ_1, γ_2 such that $\gamma_1 \cap \gamma_2 = \{0\}$. γ_1, γ_2 are supposed to be $C^{1,\alpha}$ for some $0 < \alpha < 1$, and to meet at 0 with angle (measured in Ω) θ, $0 < \theta < \pi$. We also suppose (without loss of generality, since we can always take a smaller R) that γ_i intersects ∂D_ρ in a single point for each $i = 1, 2$, $0 < \rho < R$.



Then we look at (weak) $C^{1,\alpha}(\bar{\Omega} \sim \{0\})$, solutions of the equation

$$\sum_{i=1}^{2} D_i\left(\frac{D_i u}{\sqrt{1 + |Du|^2}}\right) = H(x, u) \quad \text{on} \quad \Omega\,, \tag{0.1}$$

where H is a locally bounded measurable function on $\bar{\Omega} \times \boldsymbol{R}$.

It is assumed that a contact angle boundary condition holds; to be precise, we suppose