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CONGRUENCE CONDITIONS ON INTEGERS REPRESENTED BY TERNARY QUADRATIC FORMS

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A relationship is proved between certain integers c primitively represented by a spinor genus of integral ternary quadratic forms and integers of the type ct^2 primitively represented by individual forms in the spinor genus. This relationship is shown to explain various representation properties observed in the literature.

There are numerous examples in the literature of pairs of integral ternary quadratic forms which lie in the same genus but which primitively represent different sets of integers according to certain congruence properties. For instance, consider the forms $f = x^2 + xy + y^2 + 9z^2$ and $g = x^2 + 3(y^2 + yz + z^2)$, which are representatives of the two equivalence classes of a genus. f primitively represents an integer of the form $4t^2$, $t > 0$, if and only if $t \equiv -1 \pmod 3$, while g primitively represents such an integer if and only if $t \equiv 1 \pmod 3$ (see [12]).

Recent work of Peters [10] indicates that the above-illustrated phenomenon is a consequence of the splitting of the genus into spinor genera. Much progress has been made in the study of spinor genus representations in the work of several authors which will be cited in detail below. However, for definite forms, for which there may be many equivalence classes within a single spinor genus, there remains the problem of relating the representations by the individual forms in the spinor genus to the representations by the spinor genus as a whole. It is this problem that we address in the present paper. The main result, appearing as Theorem 2.3, shows that under certain conditions a relationship holds between integers c primitively represented by a spinor genus and integers of the type ct^2 primitively represented by the forms in the spinor genus.

The behavior discussed here is unique to ternary forms in light of two major results. First, definite forms in 4 or more variables (primitively) represent all sufficiently large integers which are (primitively) represented by their genus. For a proof of this theorem, and its history, see [1]. Extensions of this result to representations of forms by forms, and to representations by quadratic lattices over number fields, are found in [3]. Secondly, indefinite forms in 4 or more variables (primitively) represent *all* integers (primitively) represented by their genus (see e.g., [12], Theorem 53). As the equivalence class and spinor genus of such indefinite forms