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SINGULARITIES OF SOLUTIONS TO LINEAR SECOND ORDER ELLIPTIC PARTIAL DIFFERENTIAL EQUATIONS WITH ANALYTIC COEFFICIENTS BY APPROXIMATION METHODS

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Let the linear second-order elliptic partial differential equation be given in the normal form

$$\Delta^2 v + a(x, y)v_x + b(x, y)v_y + c(x, y)v = 0\ , \qquad (x, y)\in E^2$$

with real-valued coefficients that are entire functions on $\mathscr{C}^2$ and whose coefficient $c(x, y)\leq 0$ on the disk D: $x^2+y^2\leq 1$. Let the initial domain of definition of the real-valued regular solution $v=v(x, y)$ be D. A local Chebyshev approximation scheme is given by which global information is determined concerning the location of the singularities of the principal branch of the analytic continuation of v. This follows from an error analysis of best approximates taken over certain families of regular solutions whose singularities are in comp (D). The Bergman and Gilbert Integral Operator Method is utilized in this function-theoretic extension of the theorems of S. N. Bernstein and E. B. Saff; these theorems classify the singularities of analytic functions of one complex-variable via the growth in the error of Chebyshev approximations taken over rational functions of type (n, ν).

1. Introduction. The singularities of the real-valued regular (classical) solutions of the linear second-order elliptic partial differential equation

$$(1)\qquad \mathscr{L}(v) = \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + a(x, y)\frac{\partial v}{\partial x} + b(x, y)\frac{\partial v}{\partial y} + c(x, y)v = 0$$

are considered here. The real analytic coefficients continue analytically as entire functions on $\mathscr{C}^2$ when x and y continue as independent complex variables; also, the coefficient $c(x, y) \leqq 0$ on $x^2 + y^2 \leqq 1$.

Properties of the singularities of solutions to linear elliptic partial differential equations stir special interest in several areas of mathematical physics [5, 7], for example, in potential scattering. Using function theoretic methods, R. P. Gilbert and D. L. Colton [8] determined necessary and sufficient conditions concerning the location of singularities of regular solutions v in terms of corresponding information for a unique associated analytic function f of one complex-variable. Our principle interest is in global information concerning the singularities of v independent of the associate. This information appears by approximation methods that determine