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SOME EXPANSIONS INVOLVING BASIC HYPERGEOMETRIC FUNCTIONS OF TWO VARIABLES

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In this paper we obtain expansions of q-Appell type of functions of higher dimensions. These expansions are different in nature from the ones studies thus far. Transformations and reducibility of basic double hypergeometric functions are also discussed.

Burchnall and Chaundy [5, 6] has made a systematic study of the expansions of the Appell functions. Later on Jackson [8, 9] defined the q-analogue of Appell functions and made a parallel study by obtaining q-analogues of most of the results of Burchnall and Chaundy. Jackson [8; p. 78] had pointed out that it does not seem possible to obtain simple extensions of the expansions (46)-(51) of Burchnall and Chaundy [5]. In §3 of this paper we give q-analogues of five of the results, viz., (46)-(49) and (51) of Burchnall and Chaundy [5] cited above.

It may be remarked that Andrews [1] had proved that the q-analogue of Appell's function $F^{(1)}$ defined by Jackson [8] is infact reducible to the basic hypergeometric series ${}_3\phi_2$. We show in §4 that some higher dimensional analogues of double hypergeometric functions could also be reduced to basic hypergeometric series and use the reduction formula for obtaining some interesting transformations for double hypergeometric functions. In this sequal we also derive q-analogues of some of the well known transformations of Appell functions and discuss their reducibility.

2. Definitions and notations.

If we let

$$[a; q]_n = (1 - a)(1 - aq) \cdots (1 - aq^{n-1}) ,$$
$$[a; q]_0 = 1 \quad \text{and} \quad [a; q]_\infty = \prod_{r=0}^{\infty} (1 - aq^r) ,$$

then we may define the basic hypergeometric series as

$$ {}_{p+1}\phi_{p+r}\left[\begin{matrix} a_1, a_2, \cdots, a_{p+1}; q; x \\ b_1, b_2, \cdots, b_{p+r} \end{matrix}\right] = \sum_{n=0}^{\infty} \frac{[a_1; q]_n \cdots [a_{p+1}; q]_n x^n (-)^{nr} q^{rn/2(n-1)}}{[q; q]_n [b_1; q]_n \cdots [b_{p+r}; q]_n} , \qquad |q| < 1 ,$$

where the series ${}_{p+1}\phi_{p+r}(x)$ converges for all positive integral values of r and for all x, except when $r = 0$, it converges only for $|x| < 1$.