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STABLE SEQUENCES IN PRE-ABELIAN CATEGORIES

YONINA S. COOPER

In the Pacific Journal of Mathematics, 71 (1977), Richman and Walker gave a natural definition for Ext **in an arbitrary pre-abelian category. Their Theorem 4, which states that** $(\alpha E)\beta = \alpha(E\beta)$ **for an arbitrary sequence** E**, is in error. We show, however, that** $(\alpha E)\beta = \alpha(E\beta)$ **does hold for a stable exact sequence. Without Theorem 4, the crucial step in their theory is showing that** αE **is stable if** E **is stable. We prove this. Consequently, the theory of Richman and Walker for** Ext **in a pre-abelian category is valid.**

1. Introduction. An additive category with kernels and cokernels is called *pre-abelian*. Richman and Walker [3] developed an additive bifunctor Ext from an arbitrary pre-abelian category to the category of abelian groups. The Ext introduced in [3] coincides with the standard Ext (e.g., see [2]) if the category is, in fact, abelian. This theory is subsequently used by Richman and Walker [4] in the category of valuated groups. The theory of [3] is also used in [1] to examine certain relative homological algebras and to compute certain $\mathrm{Ext}(C, A)$ in the category of finite valuated groups.

However, Theorem 4 [3, p. 523] is incorrect. Without Theorem 4, one needs to prove that the sequence αE is stable if the sequence E is stable. This is our Theorem 2.

We use the terminology and notation of [3]. Thus, we are working working in an arbitrary pre-abelian category. If $f: A \to B$ and $\alpha: A \to A'$, then the pushout diagram

$$\begin{array}{ccc} A & \xrightarrow{f} & B \\ \alpha\downarrow & & \downarrow\xi \\ A' & \xrightarrow{\beta} & P \end{array}$$

is constructed by setting $P = \mathrm{coker}(f \oplus (-\alpha))\Delta$, where $\Delta: A \to A \oplus A$ is the diagonal map. We say that β is the pushout of f along α. Pullbacks are obtained dually. A *sequence* E is a diagram $A \xrightarrow{f} B \xrightarrow{g} C$ such that $gf = 0$. E is *left exact* if f is the kernel of g, *right exact* if g is the cokernel of f, and *exact* if it is both left and right exact. If $\alpha: A \to A'$, we pushout f along α to construct the sequence αE.