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LOCALLY SMOOTH TORUS GROUP ACTIONS ON INTEGRAL COHOMOLOGY COMPLEX PROJECTIVE SPACES

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Let X be a HCP^n which admits a nontrivial smooth S^1 action. Petrie defined and studied a set functions $\delta_i(m)$ which are important in the study of local representations. In this paper, we extended Petrie's result to locally smooth torus group actions on integral cohomology complex projective spaces.

Introduction. Let X be a closed smooth manifold homotopically equivalent to CP^n which admits a nontrivial smooth S^1 action. An interesting problem is to study the structure of the representations of S^1 on the normal fibers to the various components of the fixed point set. Let the fixed point set X^{S^1} consist of k connected components X_i, $i = 1, 2, \cdots, k$. Let η be the equivariant Hopf bundle [3]. Choose $x_i \in X_i$ and define k integers a_i by $\eta|_{x_i}(t) = t^{a_i}$. Then Petrie [3] proved the following.

THEOREM 0.1. *The k integers a_i are distinct.*

Suppose further that $X_i = x_i$, isolated points. Let $TX|_{x_i}(t) = \sum_{j=1}^{n} t^{x_{ij}}$. For each integer m and each $i = 1, \cdots, k$, set

$$n_j(m) = \text{number of } j \neq i \text{ such that } m \text{ divides } a_j - a_i\ ;$$
$$d_i(m) = \text{number of } j = 1, \cdots, n \text{ such that } m \text{ divides } x_{ij}\ ,$$
$$\delta_i(m) = n_i(m) - d_i(m)\ .$$

Then Petrie [3] proved the following.

THEOREM 0.2. *$\delta_i(m) \geqq 0$ and $\delta_i(p^r) = 0$ if p is a prime.*

Although so far the actions are smooth, it is not difficult to see that the numbers a_i can be defined in the same way for an S^1 action on an integral cohomology complex projective space and the numbers x_{ij} are defined if the action is locally smooth [1]. It turns out that these results are also true for locally smooth S^1 actions on integral cohomology complex projective spaces. The main purpose of this paper is to extend these results to the category of locally smooth torus actions on integral cohomology complex projective spaces such that the fixed point sets do not necessarily consist of isolated points.