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HOMOLOGY COBORDISMS OF 3-MANIFOLDS, KNOT CONCORDANCES, AND PRIME KNOTS

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There is a close relation between link concordances and homology cobordisms of 3-manifolds. Using this relationship we will prove that every closed, orientable 3-manifold is homology cobordant to an irreducible 3-manifold. Essentially the same construction will be used to prove that every knot in S^3 is concordant to a prime knot.

Kirby and Lickorish [3] have previously obtained the second result using a different construction. This answers problem 13 in Gordon [2].

Results of Schubert [6] show that the knots constructed are prime. In the second half of this paper we will generalize his results to characterize prime "generalized" cable knots. If K_1 is a knot in $S^1 \times B^2$ and K_2 is a knot in S^3, we can form the K_1 cable of K_2 by mapping $S^1 \times B^2$ into S^3, with $S^1 \times \{0\}$ going to K_2. We will show that, with an obvious restriction, the K_1 cable of K_2 is prime if and only if K_1 is prime in $S^1 \times B^2$. The final section gives a large set of examples of prime knots in $S^1 \times B^2$.

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1. Definitions and conventions. In all that follows manifolds and maps will be smooth and orientable. Intersections of submanifolds and of submanifolds with the boundary of a manifold will always be assumed to be transverse.

A *knot* K in S^3 is an embedded S^1 in S^3, a *link* of n components, L, is a collection on n disjoint knots. By $N(K)$ we indicate a tubular neighborhood of K in S^3. The *meridian* of a knot K is any embedded S^1 in $\partial N(K)$ which bounds a B^2 in $N(K)$, and which is nontrivial in $H_1(\partial N(K); Z)$. A *longitude* of K is an embedded S^1 in $\partial N(K)$ which is nontrivial in $H_1(\partial N(K); Z)$ but represents 0 in $H_1(S^3\text{-int}(N(K)); Z)$.

Two links, L_1 and L_2, each of n components are called *concordant* if there is an embedding $\bar{L}$ of n disjoint copies of $S^1 \times I$ into $S^3 \times I$, with $\bar{L}(n(S^1 \times I)) \cap S^3 \times \{0\} = L_1$ and $\bar{L}(n(S^1 \times \{I\})) \cap S^3 \times \{1\} = L_2$. Two 3-manifolds, M_1 and M_2, are *homology cobordant* if there is a 4-manifold W, with $\partial W = M_1 \cup M_2$ and the map of $H_*(M_i; Z) \rightarrow H_*(W; Z)$ an isomorphism.

A 3-manifold M is *irreducible* if every embedded S^2 in M bounds an embedded B^3. A link L is *irreducible*, or *unsplittable*, if S^3-L