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PERIODIC GAUSSIAN OSTERWALDER-SCHRADER POSITIVE PROCESSES AND THE TWO-SIDED MARKOV PROPERTY ON THE CIRCLE

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Gaussian processes in a class of stochastic processes associated with quantum systems at nonzero temperature (the periodic stochastic processes satisfying Osterwalder-Schrader (OS) positivity on the circle) are studied. A representation of the covariance function of a periodic Gaussian OS-positive process is obtained which gives a complete description of all such processes. The two-sided Markov property on the circle is studied and it is determined which periodic Gaussian OS-positive processes satisfy the two-sided Markov property on the circle. It is shown that every periodic Gaussian OS-positive process is the restriction of a periodic Gaussian two-sided Markov process. For nonperiodic Gaussian OS-positive processes it is shown that the two-sided Markov property is equivalent to the Markov property.

1. Introduction. Certain stochastic processes are associated with quantum systems (e.g., Nelson [11, 12], Simon [14], Hoegh-Krohn [3], Albeverio and Hoegh-Krohn [1], Klein [5, 6], Driessler, Landau and Perez [2]). If the quantum system is at a nonzero temperature T then the associated stochastic process is periodic with period equal to the inverse temperature $\beta = (kT)^{-1}$, where k is Boltzmann's constant (Hoegh-Krohn [3], Albeverio and Hoegh-Krohn [1], Driessler, Landau and Perez [2]).

The simplest example is the Ornstein-Uhlenbeck process which is associated with the quantum mechanical harmonic oscillator. The Gaussian process $X(t)$ indexed by the real line with mean zero and covariance $E(X(t)X(s)) = (2m)^{-1}\exp(-|t-s|m)$ with $m > 0$ (the usual Ornstein-Uhlenbeck process) is associated with the one-dimensional harmonic oscillator with frequency m (i.e., with Hamiltonian $H = 1/2(-d^2/dx^2 + m^2x^2 - m)$) at zero temperature (e.g., Simon [14]). If this harmonic oscillator is considered at a nonzero temperature T, then the associated stochastic process is the periodic Gaussian process $X_\beta(t)$ indexed by the real line with period $\beta = (kT)^{-1}$ having mean zero and covariance

$$E(X_\beta(t)X_\beta(s)) = (2m[1-\exp(-m\beta)])^{-1}(\exp(-|t-s|m) \\ + \exp(-(\beta - |t-s|)m))$$

for $|t-s| \leqq \beta$ (Hoegh-Krohn [3]). We will call $X_\beta(t)$ the periodic