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HYPERGEOMETRIC SERIES WITH A p-ADIC VARIABLE

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Hypergeometric series with a p-adic variable and ratios of such series, as originally considered by B. Dwork, are evaluated at $x=1$. Koblitz's conjecture on the limit of ratios of partial sums of hypergeometric series in the supersingular case is examined and a sufficient condition for the validity of this conjecture is given.

Introduction. In studying the zeta function of a hypersurface, B. Dwork was led to a study of ratios of p-adic hypergeometric series. In [4] and [5] he showed that under certain conditions these ratios had an analytic continuation beyond their disc of convergence. N. Koblitz has recently shown, [6], that the value of the continuation of $F(a, b; 1; x)/F(a', b'; 1; x^p)$ at $x = 1$ is $\Gamma_p(a)\Gamma_p(b)/\Gamma_p(a + b)$, where Γ_p is Morita's p-adic gamma function. Koblitz then conjectured that the ratio of the partial sums

$$F_{s+1}(a, b; 1; 1)/F_s(a', b'; 1; 1) ,$$

where

$$F_s(a, b; c; x) = \sum_{0 \leqq n < p^s} \frac{(a)_n(b)_n}{(c)_n n!} x^n$$

has a limit as s approaches infinity for all a and b except for a special case in which the ratio is 0/0. In addition, he gave an expected formula in terms of Γ_p.

In § 1 we will calculate the value of the continuation of $F(a, b; c; x)/F(a', b'; c'; x^p)$ at $x = 1$ for any appropriate a, b and c. In § 2 we will consider the value at $x = 1$ of hypergeometric series in which $c \in \Omega_p - \boldsymbol{Z}_p$ and of certain cases of generalized hypergeometric series. It will be seen, in particular, that Dixon's theorem and Saalschütz's formula hold for p-adic variables. In the last section we consider Koblitz's conjecture, generalized to allow for other c and x. While we give some examples where the conjecture is not quite true, the basic result is a condition on the size of $F_s(a', b'; c'; 1)$ which is sufficient to prove Koblitz's conjecture and its generalization to $c \neq 1$. The proof this theorem connects some of the results of § 2, where c was not in $\boldsymbol{Z}_p$, with Dwork's work, in which $c \in \boldsymbol{Z}_p$.

1. Ratios of hypergeometric functions. If a, b and c are in $\boldsymbol{Z}_p$, then the hypergeometric series