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VECTOR-VALUED FUNCTIONS AS FAMILIES OF SCALAR-VALUED FUNCTIONS

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One time-honored way of studying the properties of a vector measure F with values in a Banach space X, with dual X^* is to examine properties of the family of scalar measures $\{\langle x^*, F\rangle : x^* \in X^*, \|x^*\| \leqq 1\}$. The purpose of this paper is to undertake a similar study for vector-valued functions. The first theorem proved in this vein was the classical Pettis measurability theorem which states that if X is a separable Banach space and f is an X-valued function such that $\langle x^*, f\rangle$ is measurable for each x^* in X^*, then f is a measurable function. What we propose to do is to take a bounded function f with values in X, form the associated family $\mathscr{F} = \{\langle x^*, f\rangle : x^* \in X^*, \|x^*\| \leqq 1\}$ and study how measurability and integrability properties of f are reflected by topological properties of $\mathscr{F}$ in the spaces L_∞ and $B(\Sigma)$.

Throughout this paper (Ω, Σ, μ) is a finite measure space and X is a Banach space with dual X^*. A function $f: \Omega \to X$ is (μ-) measurable if it is the a.e. limit of a sequence of simple functions. Standard arguments show that if μ is a Radon measure, then μ-measurablity is equivalent to Lusin μ-measurability, which means that for every compact set $K \subset \Omega$ and for every number $\delta > 0$ there is a compact set $K' \subset K$ such that $\mu(K \backslash K') < \delta$ and f restricted to K' is continuous. The function f is *weakly* (μ-) *measurable* if $\langle x^*, f\rangle$ is measurable for all x^* in X^*. If X is the dual of a space Y, then f is *weak**-(μ-) *measurable* if $\langle f, y\rangle$ is measurable for all y in Y. A function $f: \Omega \to X$ is weakly equivalent to a function $g: \Omega \to X$ if $\langle x^*, f\rangle = \langle x^*, g\rangle$a.e., for every x^* in X^*. If X is the dual of space $\mathscr{Y}$, then f and g are weak*-equivalent if $\langle f, y\rangle = \langle g, y\rangle$ a.e., for all y in $\mathscr{Y}$.

A weakly measurable function $f: \Omega \to X$ is Pettis integrable if for each E in Σ there is an element $P - \int_E f d\mu$ in X such that

$$x^*\left(P = \int_E f d\mu\right) = \int_E x^* f d\mu \,.$$

1. The family $\{\langle x^*, f\rangle : \|x^*\| \leqq 1\}$ as a subset of $L_\infty(\mu)$. In this section we shall study a bounded function $f: \Omega \to X$ in terms of topological properties of the associated family $\{\langle x^*, f\rangle : \|x^*\| \leqq 1\}$ as a subset of the space $L_\infty(\mu)$.