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GENERALIZED THREE-MANIFOLDS WITH ZERO-DIMENSIONAL NONMANIFOLD SET

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This paper investigates the statement (GM): "If X is a compact generalized 3-manifold without boundary, and whose nonmanifold set is 0-dimensional, then X is the cell-like image of some closed 3-manifold." Some necessary and some sufficient conditions on X are given for (GM) to be true. The question of whether (GM) is true in general is shown to be inextricably tangled with the Poincaré Conjecture: (1) If the Poincaré Conjecture fails, then there is an acyclic, monotone union M of handlebodies whose one-point compactification $\hat{M}$ is a generalized 3-manifold, yet $\hat{M}$ is not the cell-like image of any compact 3-manifold. (2) If X is a compact generalized 3-manifold with zero-dimensional singular set S and no π_1-torsion in any sufficiently tight neighborhood of S, then (modulo the Poincaré Conjecture) X is the cell-like image of a compact 3-manifold.

1. Basic definitions and notation. In this paper a *generalized 3-manifold* (3-gm) X will be compact and without boundary, and so will be a compact, finite-dimensional absolute neighborhood retract (ANR) so that for every point $x \in X$

$$H_*(X, X - \{x\}) \cong H_*(S^3) .$$

We are using S^n to denote the unit sphere in Euclidean $(n+1)$-space $\boldsymbol{R}^{n+1}$. We will use B^n, Δ^n, $\boldsymbol{Z}$ and I to denote the unit n-ball, the standard n-simplex, the integers and the unit interval $[0, 1]$ respectively. All homologies will use $\boldsymbol{Z}$ for coefficients.

If X is a 3-gm, the *singular set* $S = S(X)$ of X will be the set of those points in X which have no neighborhood homeomorphic to $\boldsymbol{R}^3$. The set $X - S = M(X)$, called the *manifold set* of X, will be a noncompact 3-manifold without boundary if S is neither empty nor all of X. In this paper we will be concerned only with those 3-gms whose singular set is 0-dimensional. As of this writing, the status of (GM) for those 3-gms whose singular set has dimension greater than 0 is not known. For solutions to the analogous problems in dimensions five and higher, see [6] and [18].

A *cell-like* map is a surjection in which the inverse image of each point is cell-like. (All "maps" are continuous.) A continuum (compact, connected set) in an ANR is *cell-like* if it contracts in each neighborhood of itself. If X is a 3-gm, then we say that X *resolves*, or "admits a resolution," if there is a cell-like map from a