# TENSOR PRODUCTS FOR $\left.S L_{2}{ }^{( } \mathfrak{f}\right) \mathrm{II}$, SUPERCUSPIDAL REPRESENTATIONS 

C. Asmuth and J. Repka


#### Abstract

Certain pairs of quadratic extension Weil representations of $S L_{2}(f)$ have as their tensor product the quaternion Weil representations. This fact is used to develop a method for decomposing tensor products of certain pairs of irreducible supercuspidal representations of $S L_{2}(\mathrm{f})$.


1. The object of this paper is to give decompositions of tensor products of certain pairs of supercuspidal representations of $S L_{2}(\mathfrak{f})$ where $\mathfrak{f}$ is a $p$-adic field of odd residual characteristic. These tensor products are summands of the quaternion Weil representation. The second section includes preliminaries concerning the quaternion Weil representation and its relation to quadratic extension Weil representations.

The third section sets up the basic mechanism by which the tensor product summands in the quaternion Weil representation are analyzed. It ends with what is the central theorem of the paper. This theorem provides information on decompositions of tensor products in terms of characters of certain multiplicative subgroups of the quaternions.

The fourth section is a catalogue of data on characters of multiplicative subgroups of the quaternions. It is based on [3] and to an extent on [5]. Unifortunately, the work in [5] excludes the cases needed here. For that reason I would like to particularly thank L. Corwin for a manuscript version [4] which includes some specific computations for the quaternion case. The computations in [4] and [5] are similar.

The fifth section gives the decompositions of tensor products explicitly. The main result of $\S 3$ and the data in $\S 4$ combine to produce the end results.

The sixth section is independent of the others. It gives a (brief) description of how these and other results ([6] and [7]) can be used to give partial results for the tensor products of pairs of supercuspidal representations not covered in the above work. Specifically, we can describe which tensor products contain a continuous part in their decompositions, and give the multiplicities explicitly. We can also give the multiplicities for some of the discrete components.
2. Let $f$ be a $p$-adic field with odd residual characteristic. Let

