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QUASIDIAGONAL WEIGHTED SHIFTS

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We characterize the quasidiagonality of a two-way weighted shift solely in terms of the weights. We use the characterization to show that quasidiagonality fails to be an invariant for similarity: an operator similar to the (unweighted) bilateral shift may fail to be quasidiagonal.

Introduction. We given necessary and sufficient conditions for the quasidiagonality of a two-way weighted shift. We use the characterization to show that quasidiagonality fails to be invariant for similarity.

A (bounded) operator A on a separable Hilbert space is *quasidiagonal* if there exists a sequence $\{P_n\}_1^\infty$ of orthogonal projections of finite rank such that $\{P_n\}$ converges strongly to 1 and $\{P_nA - AP_n\}$ converges uniformly to 0. The only facts about quasidiagonal operators that we use in this paper are that each normal operator is quasidiagonal and that each compact perturbation of a quasidiagonal operator is again quasidiagonal. Both facts are proved in [6, § 4], where the concept was introduced.

Throughout this paper $\{e_i\}_{-\infty}^{+\infty}$ is a fixed basis for a complex Hilbert space, and $\{w_i\}_{-\infty}^{+\infty}$ is a fixed, bounded sequence of complex numbers. The operator B determined on the Hilbert space by the equations $Be_i = w_ie_{i+1}$ is a *two-way weighted shift* with weight sequence $\{w_i\}_{-\infty}^{+\infty}$. The main goal of the paper is to prove Theorems 1, 2, and 5, which together characterize quasidiagonal weighted shifts solely in terms of their weight sequences.

1. Sufficiency.

THEOREM 1. *If the weight sequence of a two-way weighted shift has 0 as a limit point in both directions, then it is quasidiagonal.*

Proof. Let $\{w_{i_n}\}_{n=-\infty}^{+\infty}$ be a subsequence of the weights that converges to 0 in both directions. For each positive integer n, define P_n to be the (orthogonal) projection onto the span of $\{e_{i_{-n}+1}, e_{i_{-n}+2}, \cdots, e_{i_n}\}$. It is easy to compute that $\|P_nB - BP_n\|$ is the larger of $|w_{i_{-n}}|$ and $|w_{i_n}|$; therefore $\|P_nB - BP_n\| \to 0$. The P_n are increasing, and the union of their ranges contains all the vectors in the basis; therefore $P_n \to 1$ strongly. By definition, consequently, B is quasidiagonal. □