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FINITE HANKEL TRANSFORMS OF DISTRIBUTIONS

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Finite Hankel transforms of the second and third kind of distributions are defined and inversion theorems are established in the distributional sense. Operational transform formulae are obtained for the both transforms. These are applied to solve certain partial differential equations with distributional boundary conditions.

1. **Introduction.** Finite Hankel transforms of classical functions were first introduced by Sneddon [8] who applied them in solving boundary value problems for systems possessing axial symmetry. There are three kinds of finite Hankel transforms depending upon the nature of the kernel involved. These are associated to the three kinds of expansions of an arbitrary function, viz. Fourier Bessel series [6], Dini series [6] and series involving cross products of Bessel functions [4] respectively.

Finite Hankel transforms of distributions were given by Zemanian [11], Pandey and Pathak [3] as special cases of their work on general eigenfunction expansion of distributions. But, Dube [1] studied finite Hankel transform of the first kind of distributions independently. To get a deep insight it is necessary to study the other two transforms also independently. In [12] and [3] the inversion theorems are given without any consideration of the values of $H+\nu$ occurring in the definition of the transform (see (4.4)), where as the classical Dini series involves a term depending upon it. This motivated us to study independently the finite Hankel transforms of the second and third kind of distributions.

The present paper is divided into two parts. In the first part we extend the classical inversion theorem for finite Hankel transform of the second kind [6, p. 601] to a class of distributions, which gives rise to the Dini expansion of the distributions. The series converges in the weak distributional sense. We derive an operational transform formula which together with inversion formula is applied in solving certain distributional differential equations. In the second part of the paper we extend the inversion theorem for finite Hankel transform of the third kind [4] to a class of distributions. Here also the series converges in the weak distributional sense. Finally we give an application of the finite Hankel transform of the third kind.

2. **The notation and terminology.** We follow the notation