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FIXED POINT CLASSES OF A FIBER MAP

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Let (E, p, B) be a fiber space with E, B and all fibers compact connected ANR's. Let $f: E \to E$ be a fiber map, then f induces $\bar{f}: B \to B$. For each fixed point b of $\bar{f}$, we define $f_b = f|p^{-1}(b): p^{-1}(b) \to p^{-1}(b)$. Then $p \circ f = \bar{f} \circ p$ and $i_b \circ f_b = f \circ i_b$, where i_b is the inclusion map. We have Nielsen numbers $N(f)$, $N(\bar{f})$ and $N(f_b)$. A product formula relating these Nielsen numbers was published by Brown in 1967. There have been several improvements of the formula since that time.

In this paper, we study the structure of the fixed point classes of f, and prove some theorems about the product formula of the Nielsen number of a fiber map, which imply results of Fadell and of Pak.

Throughout this paper we assume all spaces are path-connected and all fiber spaces are Hurewicz fiber spaces.

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1. Fixed point H-classes. The concept of fixed point H-classes is presented in [8] and [7]. For convenience of calculation, we give its definition a precise algebraic formulation.

Let X be a space, and H be a normal subgroup of $\pi_1(X)$ (which means that for each $x \in X$, a normal subgroup $H(x)$ of $\pi_1(X, x)$ is defined, such that for any path w in X from x to x', we have $w_*(H(x)) = H(x')$, where $w_*: \pi_1(X, x) \to \pi_1(X, x')$ is defined by $w_*(\langle a \rangle) = \langle w^{-1}aw \rangle$, for any $\langle a \rangle \in \pi_1(X, x)$). Two paths c, d in X are said to be *H-homotopic* and written $c \overset{H}{\simeq} d$, if $c(0) = d(0)$, $c(1) = d(1)$ and $\langle cd^{-1} \rangle \in H$. One can easily see that when $c \overset{H}{\simeq} d$, then $d \overset{H}{\simeq} c$, $c^{-1} \overset{H}{\simeq} d^{-1}$ and also $uc \overset{H}{\simeq} ud$, $cv \overset{H}{\simeq} dv$ if uc and cv are well-defined.

Let $x \in X$, we can think of every element of $\pi_1(X, x)/H(x)$ as a $\overset{H}{\simeq}$ equivalence class of loops based at x. Let $\langle a \rangle_H$ denote the $\overset{H}{\simeq}$ equivalence class of the loop a. For each path w in X from x to x', let $w_H: \pi_1(X, x)/H(x) \to \pi_1(X, x')/H(x')$ be the homomorphism induced by w_*, that is, $w_H(\langle a \rangle_H) = \langle w^{-1}aw \rangle_H$.

Suppose that a map $f: X \to X$ satisfies $f_\pi(H) \subset H$ (it means that for any $x \in X$, $f_\pi(H(x)) \subset H(f(x))$ where $f_\pi: \pi_1(X, x) \to \pi_1(X, f(x))$ is the induced homomorphism). Then, for each $x \in X$, $f_\pi: \pi_1(X, x) \to \pi_1(X, f(x))$ induces a homomorphism $f_H: \pi_1(X, x)/H(x) \to \pi_1(X, f(x))/$