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SHIFTS ON INDEFINITE INNER PRODUCT SPACES II

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This paper continues the study of isometries on indefinite inner product spaces by means of their wandering subspaces. In the author's earlier paper of the same title (Pacific J. Math., 81 (1979), 113–130), it was shown that the subspace on which an isometry acts as a shift need not be regular and that vectors in this subspace need not be recoverable from their Fourier coefficients by summation. We present here necessary and sufficient conditions for this situation not to occur, and also show that these conditions are sufficient (but not necessary) for the isometry to have a Wold decomposition.

1. Introduction. Throughout this paper we will be using the notation and assuming the results of the paper [4]. Our attention will be restricted to isometries on *Krein* spaces $\mathscr{K}$ (see [1, Chapter V]), where the indefinite inner product $[\cdot, \cdot]$ is related to a Hilbert space inner product $(\cdot, \cdot)$ on $\mathscr{K}$ by means of a *fundamental symmetry* J:

$$[x, y] = (Jx, y) , \quad J = J^* = J^{-1} .$$

Except in § 2, where we prove a lemma on projections in Hilbert space, we will be using the indefinite inner product $[\cdot, \cdot]$ to define properties of operators and subspaces. In particular, an isometry V preserves the indefinite inner product, and the concepts of adjoint and orthogonality use this inner product. Thus if $\mathscr{L}$ is a subspace of $\mathscr{K}$, then

$$\mathscr{L}^{\perp} = \{h \in \mathscr{K} : [h, k] = 0 \text{ for all } k \in \mathscr{L}\} .$$

If $\mathscr{L} \oplus \mathscr{L}^{\perp} = \mathscr{K}$, then $\mathscr{L}$ is called *regular*. A projection P satisfies $P = P^2 = P^*$, i.e., self-adjoint with respect to the indefinite inner product, and the regular subspaces are those that are the ranges of projections. In § 2, where an indefinite inner product will not be used, we will use Q to denote an orthogonal projection in Hilbert space and P to denote any other projection.

Suppose V is an isometry on a Krein space $\mathscr{K}$, and let $\mathscr{L} = (VK)^{\perp}$. Then $\mathscr{L}$ is wandering for V, i.e., $V^p\mathscr{L} \perp V^q\mathscr{L}$ for all nonnegative integers $p \neq q$. Since V is an isometry, VV^* is the projection onto $V\mathscr{K}$. Thus the projection P onto $\mathscr{L}$ is given by $P = I - VV^*$, and so $\mathscr{L}$ is regular.

We make the definition $M_+(\mathscr{L}) = \bigvee_{n=0}^{\infty} V^n\mathscr{L}$. Every vector