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EXTENDING FUNCTIONS FROM PRODUCTS WITH A METRIC FACTOR AND ABSOLUTES

TEODOR C. PRZYMUSIŃSKI

Extendability of continuous functions from products with a metric or a paracompact p-space factor is studied. We introduce and investigate completions mX and pX of a completely regular space X defined as "largest" spaces Y containing X as a dense subspace such that every continuous real-valued function extends continuously from $X \times Z$ over $Y \times Z$ where Z is a metric or a paracompact p-space, respectively. We study the relationship between mX (resp. pX) and the Hewitt realcompactification υX (resp. the Dieudonné completion μX) of X. We show that for normal and countably paracompact spaces $mX = \upsilon X$ and $pX = \mu X$, but neither normality nor countable paracompactness alone suffices. The relationship between completions mX and pX and the absolute EX of X is discussed.

1. Introduction. All spaces are completely regular and all functions and mappings are continuous. Symbols $\boldsymbol{F}, \boldsymbol{M}, \boldsymbol{C}$ and $\boldsymbol{P}$ denote classes of finite spaces, metrizable spaces, compact spaces and paracompact p-spaces, respectively. We recall that X is a *paracompact p-space* if it is a closed subspace of a product space $M \times C$, where M is metrizable and C is compact or—equivalently—if X is an inverse image of a metrizable space under a perfect mapping. For all undefined notions the reader is referred to [3].

Let X be a subspace of a space Y and let τ be a cardinal number. We recall the definition of *P^{τ}-embedding* of X in Y. Our definition is equivalent to the original definition of this notion involving the extendability of continuous pseudometrics [see [10] for the proof and for more information].

If τ is infinite, then X is P^{τ}-embedded in Y if every mapping $f: X \to B$ of X into a Banach space B of weight τ can be continuously extended over Y. If τ is finite, then X is P^{τ}-embedded in Y if X is C^*-embedded in Y. Moreover, X is *P-embedded* in Y if X is P^{τ}-embedded in Y for every τ. It is known that $P^{\aleph_0}$-embedding is equivalent to C-embedding [4]. The following theorem gives a product-theoretic characterization of P^{τ}-embedding. ($X \subset_{C^*} Y$ means that X is C^*-embedded in Y, etc.)

THEOREM 0 ([8], [10]). *For a subspace X of Y and a cardinal number τ the following are equivalent*:

(i) $X \subset_{P^{\tau}} Y$;