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ON SEMISIMPLE RINGS THAT ARE CENTRALIZER NEAR-RINGS

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Let G be a finite group with identity 0 and let $\mathscr{A}$ be a group of automorphisms of G. The set $C(\mathscr{A}; G) = \{f: G \to G \mid f(0) = 0,\ f(\gamma v) = \gamma f(v)$ for every $\gamma \in \mathscr{A},\ v \in G\}$ is the centralizer near-ring determined by $\mathscr{A}$ and G. In this paper we consider the following "representation" questions: (I) Which finite semisimple near-rings are of $C(\mathscr{A}; G)$-type? and (II) Which finite rings are of $C(\mathscr{A}; G)$-type?

1. Introduction. Let G be a finite group and let Γ denote a semigroup of endomorphisms of G. The set of functions $C(\Gamma; G) = \{f: G \to G \mid f(0) = 0$ and $f(\gamma v) = \gamma f(v)$ for every $\gamma \in \Gamma,\ v \in G\}$ forms a zero-symmetric near-ring under function addition and function composition. (Since all near-rings in this paper will be zero-symmetric this adjective will henceforth be omitted.) Such "centralizer near-rings" are indeed general, for it is shown in [7] that if N is any near-ring (with identity) then there exists a group G and a semigroup of endomorphisms Γ such that $N \cong C(\Gamma; G)$.

The structure of centralizer near-rings has been studied for various G's and Γ's, e.g. when $\Gamma = \mathscr{A}$ is a group of automorphisms of a finite group G ([5]), or when Γ is a finite ring with 1 and G is a faithful, unital Γ-module ([6]). From a structure theorem due to Betsch [1] we have that a finite near-ring N, which is not a ring, is simple if and only if $N \cong C(\mathscr{A}; G)$ where $\mathscr{A}$ is a fixed point free group of automorphisms of a finite group G. (A group $\mathscr{A}$ of automorphisms is fixed point free if the identity map in $\mathscr{A}$ is the only element of $\mathscr{A}$ that fixes a nonidentity element of G.)

Since every finite simple nonring is of "$C(\mathscr{A}; G)$-type" it is natural to ask for which finite near-rings does there exist a finite group G and a group of automorphisms $\mathscr{A}$ such that $N \cong C(\mathscr{A}; G)$, i.e. which finite near-rings are of $C(\mathscr{A}; G)$-type? In this paper we restrict our attention to the following more specific questions.

I. Which finite semisimple near-rings are of $C(\mathscr{A}; G)$-type?

II. Which finite rings are of $C(\mathscr{A}; G)$-type?

It will become clear in this paper that the "centralizer representation" problems I and II give rise to nontrivial group-theoretic, combinatoric problems.

In providing partial solutions to problems I and II we show that certain semisimple near-rings are not of $C(\mathscr{A}; G)$-type. Moreover