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ON COMPACTIFICATIONS OF METRIC SPACES WITH TRANSFINITE DIMENSIONS

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In this paper we prove that every separable metric space X with transfinite dimension $\operatorname{Ind} X$ has metric compactification cX such that

$$\operatorname{Ind} cX = \operatorname{Ind} X\,, \quad \operatorname{ind} cX = \operatorname{ind} X\,, \quad D(cX) = D(X)\,,$$

where $\operatorname{ind} X(\operatorname{Ind} X)$ denotes small (large) inductive transfinite dimension, and $D(X)$ denotes the transfinite D-dimension. More generally, let T be a set of invariants (ind, Ind, D). We consider the following problem:

Let $R \subseteqq T$ and X be a metric space. Does there exist a bicompactum (complete space) $cX \supset X$ such that

$$\mu(X) = \mu(cX) \quad \textbf{for} \quad \mu \in R\,.$$

When it is not so, we give counterexamples. We give also necessary and sufficient conditions of the existence of transfinite dimensions of separable metric space in terms of compactifications.

0. Introduction. In this paper we consider three transfinite invariants: $\operatorname{ind} X$, $\operatorname{Ind} X$, $D(X)$ where $\operatorname{ind} X$ (respectively, $\operatorname{Ind} X$) is small (respectively large) transfinite inductive dimension and $D(X)$ is D-dimension, see [3], Henderson.

DEFINITION 0.1. (a) $\operatorname{ind} X = -1 \Leftrightarrow X = \varnothing$.

(b) We assume that for every ordinal number $\alpha < \beta$ the class of spaces X with $\operatorname{ind} X \leqq \alpha$ is defined. Then $\operatorname{ind} X \leqq \beta$ if for every point $x \in X$ and a closed subset F, $x \notin F \subset X$, there exists a neighborhood O_x of x such that

$$O_x \subset X \backslash F$$
$$\operatorname{ind} FrO_x \leqq \alpha < \beta_1\,.$$

We put $\operatorname{ind} X = \min\{\beta : \operatorname{ind} X \leqq \beta\}$.

(c) The dimension $\operatorname{ind}_x X$ of a space X in a point $x \in X \leqq \beta$ if there exists a base $\{O_\lambda, \lambda\}$ in this point, such that

$$\operatorname{ind} FrO_\lambda < \beta\,.$$

We put $\operatorname{ind}_x X = \min\{\beta : \operatorname{ind}_x X \leqq \beta\}$.

[1] FrA denotes the boundary of A.