PACIFIC JOURNAL OF MATHEMATICS
Vol. 101, No. 2, 1982

CERTAIN TRANSFORMATIONS OF BASIC HYPERGEOMETRIC SERIES AND THEIR APPLICATIONS

V. K. JAIN

We obtain identities of Rogers-Ramanujan type related to the modulus 13. We also obtain the q-analogues of the nearly-poised summation theorems and use them for obtaining q-analogues of general transformations of nearly-poised hypergeometric series. We also discuss some important applications of the transformations obtained in this note.

Recently, Askey and Wilson [4] derived the transformation

$$ {}_4\phi_3\begin{bmatrix} a^2, b^2, c, d; q; q \\ ab\sqrt{q}, \ -ab\sqrt{q}, \ -cd \end{bmatrix} = {}_4\phi_3\begin{bmatrix} a^2, b^2, c^2, d^2; q^2; q^2 \\ a^2b^2q, \ -cd, \ -cdq \end{bmatrix}, \tag{1.1} $$

(provided a, b, c, or d is of the form q^{-N}, N a nonnegative integer). In an earlier paper [11] we have an alternative proof of (1.1). We begin this note by showing in § 3 that all the transformations proved by Singh [13], for obtaining the q-analogues of identities of the Cayley-Orr type, can be deduced from (1.1). We also show that (1.1) may be used effectively to prove the following transformation:

$$ \begin{aligned} &{}_8\phi_6\begin{bmatrix} a, q\sqrt{a}, -q\sqrt{a}, iq^{-n}, -iq^{-n}, -q^{-n}, q^{-n}, o; q; -aq^{1+4n} \\ \sqrt{a}, -\sqrt{a}, -iaq^{1+n}, iaq^{1+n}, -aq^{1+n}, aq^{1+n} \end{bmatrix} \\ &\qquad = \frac{[aq; q]_{4n}}{[a^4q^{4+4n}; q^4]_n}\, {}_3\phi_2\begin{bmatrix} -q^{-2n}, q^{-2n}, o; q^2; q^2 \\ q^{-4n}/a, q^{1-4n}/a \end{bmatrix}, \end{aligned} \tag{1.2} $$

due to Andrews [2] which is his key result for obtaining the identities of the Rogers-Ramanujan type of modulus 11. In fact, we shall prove the transformation:

$$ \begin{aligned} &{}_8\phi_7\begin{bmatrix} a, q\sqrt{a}, \ -q\sqrt{a}, c, e, -e, \ -q^{-n}, \ q^{-n}; q; \dfrac{a^2q^{2+2n}}{ce^2} \\ \sqrt{a}, -\sqrt{a}, aq/c, aq/e, -aq/e, \ -aq^{1+n}, aq^{1+n} \end{bmatrix} \\ &\qquad = \frac{[a^2q^2; q^2]_n[-aq/e^2: q]_{2n}}{[a^2q^2/c^2; q^2]_n[-aq; q]_{2n}} \\ &\qquad\quad \times \frac{[a^2q^2/c^2e^2; q^2]_n e^{2n}}{[a^2q^2/e^2; q^2]_n}\, {}_4\phi_3\begin{bmatrix} \dfrac{ce^2}{a^2}q^{-2n-1}, \dfrac{ce^2}{a^2}q^{-2n}, \ e^2, \ q^{-2n}; q^2; q^2 \\ \dfrac{e^2}{a^2}c^2q^{-2n}, \ -\dfrac{e^2}{a}q^{-2n}, \ -\dfrac{e^2}{a}q^{1-2n} \end{bmatrix} \end{aligned} \tag{1.3} $$