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APPROXIMATING CELLULAR MAPS BETWEEN LOW DIMENSIONAL POLYHEDRA

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A compact subset X of a polyhedron P is cellular in P if there is a pseudoisotopy of P shrinking precisely X to a point. A proper surjection between polyhedra $f: P \to Q$ is cellular if each point inverse of f is cellular in P. It is shown that if $f: P \to Q$ is a cellular map with either (i) dim $P \leqq 3$, or (ii) dim $Q \leqq 3$, then f is approximable by homeomorphisms.

Introduction. As a generalization of the concept of cellularity in a manifold, J. W. Cannon proposed in [3] that a set X in a polyhedron P be called cellular if X is compact and there is a pseudoisotopy of P which shrinks precisely X. He then defined a cellular map between polyhedra P and Q to be a proper surjection $f: P \to Q$ such that for each $q \in Q$, $f^{-1}(q)$ is cellular in P. Cannon first asked if a cellular map f is approximable by homeomorphisms when either P or Q is an n-manifold, $n \neq 4$. He conjectured that an affirmative solution to that question would lead to a solution of the more general problem of approximating cellular maps between arbitrary polyhedra. It was shown in [6] that if P or Q is an n-manifold, $n \neq 4$, then $f: P \to Q$ is approximable by homeomorphisms. Here we prove that if $\dim P \leqq 3$ or $\dim Q \leqq 3$, then f is approximable by homeomorphisms. This, then, can be viewed as an extension of the approximation theorem of Armentrout [1].

While the proof of the approximation theorem given here relies in many cases on the techniques used by Handel [5], it should be pointed out that the type of map considered by Handel is more restrictive than those considered here and in [6].

The reader is encouraged to read at least §§ 1 and 2 of [6] to gain an understanding of the stratification and cellular sets being used here before reading this paper.

1. Definitions and background. A *polyhedron* P is a subset of some Euclidean space $\boldsymbol{R}^n$ such that each point $b \in P$ has a neighborhood $N = bL$, the join of b and a compact subset L of P. Throughout, P and Q will denote polyhedra. A homotopy $H_t: P \to P$ for which H_t, $0 \leqq t < 1$, is a homeomorphism is a *pseudoisotopy*. A compact subset X of P is *cellular* in P if there is a pseudoisotopy $H_t: P \to P$ such that X is the only nondegenerate point preimage of H_1. A proper surjection $f: P \to Q$ is a *cellular map* if for each