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EXPONENTIAL DIOPHANTINE EQUATIONS

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CONTENTS

We study equations in which the unknowns are the exponents. (Work in this field originated with C. Størmer and D. H. Lehmer. More recently, Leo J. Alex has extended their results; his work relates to classification of nonabelian simple groups.)

(i) For the equation $k+7^a=3^c+5^d$, $k=3^b, 5^b, 13^b$, or 17^b, and for many similar 4-term equations, we find all integral solutions.

(ii) We find all integral solutions of $3^a+7^b=3^c+5^d+2$.

(iii) We prove that there are infinitely many odd m such that $m^a+7^b=3^c+5^d$ has only the solutions $(a, b, c, d) = (0, 0, 0, 0)$, $(0, 1, 1, 1)$.

1. Introduction. By an *exponential Diophantine equation* (*eDe*) we mean an equation in which the bases are (given or unknown) integers; the exponents are unknown integers. In this article the exponents are nonnegative as well. Examples are the equations

$$1 + 2^a + 7^b = 3^c + 5^d ,$$
$$3^a + 7^b = 3^c + 5^d + 2 .$$

(All solutions of these equations are determined in §6 of this article.)

Isolated examples of *eDe*'s occur very early in the history of the theory of numbers (Mersenne, Fermat). The equation $x^y = y^x$ is another hoary example. To compute accurately the logarithms of primes, Størmer [12] found all solutions of the equation $1 + 2^a3^b5^c = 2^d3^e5^f$; his method depended on the theory of Pell equations. Lehmer [8] refined Størmer's methods. This approach is still useful for computing the logarithm of a prime to high precision.