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DIFFERENTIABLE CURVES OF CYCLIC ORDER FOUR

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It is shown that there are only fourteen possible differentiable closed curves of cyclic order four in the real conformal plane. This classification is made with respect to numbers and types of singular points. In this regard the characteristic of a differentiable interior point of an arc and a known result of P. Erdös are used extensively.

In [5] N. D. Lane and P. Scherk introduced the characteristic of a differentiable interior point of an arc or curve in the real conformal plane. Using the notion of the characteristic, this paper classifies all possible simple differentiable curves of cyclic order four in the conformal plane in regard to the number and type of singular points. It is well known that such curves contain at most four singular points [9].

Moreover, differentiable curves of cyclic order four contain at most one double-point d and in this case each of the loops separated by d contain exactly one singular point with the characteristic (1, 1, 2) or $(1, 1, 2)_0$; cf. § 5.

In [3] C. Juel gave a similar study of such curves deriving his results using a correspondence principle dependent upon a continuous function theorem. This paper is a refinement of the Juel manuscript deriving the key global result by combining a known theorem of P. Erdös with a detailed discucssion of point osculating circles and the characteristic of such points.

In § 6 a list of all possible differentiable curves of order four is given.

1. Preliminaries.

1.1. A point p on an arc $\mathscr{A}$ is said to be (conformally) *differentiable* [4] if it satisfies two conditions:

I. For every point $R \neq p$ and for every sequence of points $s \to p$ on $\mathscr{A}$ there exists a circle C_0 such that $C(s, p, R) \to C_0$. C_0 is called the *tangent circle* of $\mathscr{A}$ at p through R and is denoted $C(p^2, R)$.

II. If $s \to p$ on $\mathscr{A}$ there exists a circle $C(p^3)$ such that $C(p^2, s) \to C(p^3)$. $C(p^3)$ is called the *osculating circle* of $\mathscr{A}$ at p. $C(p^3)$ may be the point circle p.

A point p on $\mathscr{A}$ is said to be *strongly differentiable* if the following are satisfied: