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AUTOMORPHISMS AND NONSELFADJOINT CROSSED PRODUCTS

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We are interested in the invariant subspace structure of the nonselfadjoint crossed product determined by a finite von Neumann algebra M and a trace preserving automorphism α. In this paper we investigate the form of two-sided invariant subspaces for the case that α is ergodic on the center of M.

1. **Introduction.** In this paper, we consider the typical finite maximal subdiagonal algebras which are called nonselfadjoint crossed products. These algebras are constructed as certain subalgebras of crossed products of finite von Neumann algebras by trace preserving automorphisms. Recently, McAsey, Muhly and the author studied the invariant subspace structure and the maximality of these algebras (cf. [4], [5], [6], [7]).

Let M be a von Neumann algebra with a faithful normal tracial state τ and let α be a *-automorphism of M such that $\tau\circ\alpha = \tau$. We regard M as acting on the noncommutative Lebesgue space $L^2(M, \tau)$ (cf. [10]) and consider the Hilbert space

$$L^2 = \{f: Z \longrightarrow L^2(M, \tau) \mid \Sigma \|f(n)\|_2^2 < \infty\}$$

which may be identified with $l^2(Z) \otimes L^2(M, \tau)$. Let $\mathfrak{L}$ (resp. $\mathfrak{R}$) be the left (resp. right) crossed product of M and α, and let $\mathfrak{L}_+$ (resp. $\mathfrak{R}_+$) be the left (resp. right) nonselfadjoint crossed product of $\mathfrak{L}$ (resp. $\mathfrak{R}$) (cf. §2). In [6], we showed that the following three conditions are equivalent; (i) M is a factor; (ii) a conditioned form of the Beurling-Lax-Halmos theorem is valid; and (iii) $\mathfrak{L}_+$ is a maximal σ-weakly closed subalgebra of $\mathfrak{L}$. Furthermore, in [7], we proved that α fixes the center $\mathfrak{Z}(M)$ of M elementwise if and only if the Beurling-Lax-Halmos theorem is valid. However, if α does not fix the center $\mathfrak{Z}(M)$ of M elementwise, then the form of invariant subspace is very complicated. Considering the reduction theory with respect to the abelian subalgebra $\{z \in \mathfrak{Z}(M): \alpha(z) = z\}$ of $\mathfrak{Z}(M)$, it seems to be sufficient to investigate the case that α is ergodic on $\mathfrak{Z}(M)$. Therefore, our aim in this paper is to study the invariant subspace structure of L^2 when α is ergodic on $\mathfrak{Z}(M)$. We now suppose that α is ergodic on $\mathfrak{L}(M)$. Then every two-sided invariant subspace of L^2 which is not left-reducing is left-pure, left-full, right-pure and right-full (Theorems 3.2 and 4.5). Further, if $\mathfrak{L}$ is a factor, then every proper two-sided invariant subspace of L^2 is of