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p-HENSELIAN FIELDS: K-THEORY, GALOIS COHOMOLOGY, AND GRADED WITT RINGS

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For a field F with a p-Henselian valuation v, direct sum decompositions will be proved for Milnor's K-theory mod n (n a power of the prime p), for the Galois cohomology of F with $\mathbf{Z}_n$-coefficients, and for the graded Witt ring of quadratic forms of F (with $p = 2$). In each case, the summands of the ring associated to F are copies of the corresponding ring associated to the residue field of v, and the number of summands is determined by its value group. The theorems generalize results known for a field with a complete discrete valuation.

The direct decompositions in K-theory, in Galois cohomology, and for the graded Witt ring, for a field with a complete discrete valuation are a familiar part of the "local" machinery of field theory. In view of the increasing importance of Henselian fields, it seems worthwhile to spell out just how these results for complete discrete fields generalize to the Henselian case. While such generalizations are not surprising, and may in certain cases be known to some, they have not appeared in the literature. (The Witt ring of a Henselian field has been described, see [**15**, §12.2], but not the graded Witt ring.)

The basic setting for our results is a field F with a p-Henselian valuation (p a prime number), as described in §1. The p-Henselian property is a weaker relative version of the Henselian condition on a valuation. We work with p-Henselian valuations because they are exactly the ones for which direct sum decompositions exist (at least when F has enough roots of unity) — see (2.3), (3.10), and (4.7). We will consider K-theory, cohomology, and the graded Witt ring in separate and largely independent sections. While the direct sum formulas are strikingly similar in each category, the methods used to obtain them are quite different.

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1. p-Henselian fields and a ring construction. We will use the notation (F, v, Γ) for a field F with valuation $v\colon \dot{F} \to \Gamma$ (where $\dot{F} = F - \{0\}$). The value group Γ will be written additively. The valuation ring, maximal ring, group of units, and residue field associated to v will be denoted respectively V_v, $\mathfrak{m}_v$, U_v, and $\bar{F}$. For $a \in V_v$, $\bar{a}$ will denote its image in $\bar{F}$.