PACIFIC JOURNAL OF MATHEMATICS
Vol. 106, No. 1, 1983

RENORMING AND THE THEORY OF PHI-ACCRETIVE SET-VALUED MAPPINGS

DAVID J. DOWNING AND WILLIAM O. RAY

Let X and Y be Banach spaces, ϕ: $X \to Y^*$ and P: $X \to 2^Y$; P is said to be strongly ϕ-accretive if there exists $c > 0$ so that $(w - v, \phi(x - y)) \geq c\|x - y\|^2$ whenever $x, y \in X$ and $w \in Px$, $v \in Py$. Such mappings constitute a simultaneous generalization of monotone mappings (when $Y = X^*$) and accretive mappings (when $Y = X$). By applying a fixed point theorem of J. Caristi, it is shown that if P is strongly ϕ-accretive in a localized sense and if Y can be appropriately renormed, then, under suitable continuity and range restrictions, P is an open mapping. The results generalize a number of known theorems and indicate a firm connection between the theory of ϕ-accretive mappings and the renorming characteristics of the space Y.

1. Introduction. Let X and Y be Banach spaces with Y^* the dual of Y, and let ϕ: $X \to Y^*$ be a mapping such that

(1.1) $$\phi(X) \text{ is dense in } Y^*$$

(1.2) $$\text{for each } x \in X \text{ and each } \xi \geq 0,\ \|\phi(x)\| \leq \|x\| \text{ and } \phi(\xi x) = \xi\phi(x).$$

A mapping P from X to Y is said to be *strongly ϕ-accretive* (e.g. [**1**] or [**14**]) if there is a constant $c > 0$ such that, for $x, u \in X$

$$(Px - Pu, \phi(x - u)) \geq c\|x - u\|^2.$$

The ϕ-accretive mappings were introduced in an effort to unify the theories for monotone mappings (when $Y = X^*$) and for accretive mapping (when $Y = X$). While the theorems obtained for the monotone and accretive mappings are similar in character, the methods employed are technically distinct and the goal in the study of ϕ-accretive operators is to develop a methodology which is applicable to both classes of mappings. Fundamental progress in this direction has been realized by F. E. Browder (e.g. [**1**]—[**4**]); one of his basic results in Theorem B below.

THEOREM B ([**4**]). *Let X and Y be Banach spaces with P: $X \to Y$ a strongly ϕ-accretive mapping. Suppose that one of the following two additional hypotheses holds*:

(I) *Y^* is uniformly convex and P is locally lipschitzian.*