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CYCLIC GROUPS OF AUTOMORPHISMS OF COMPACT NON-ORIENTABLE KLEIN SURFACES WITHOUT BOUNDARY

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We obtain the minimum genus of the compact non-orientable Klein surfaces of genus $p \geq 3$ without boundary which have a given cyclic group of automorphisms.

1. Introduction. Let X be a compact Klein surface [**1**]. Singerman [**8**] showed that the order of a group of automorphisms of a surface X without boundary of algebraic genus $g \geq 2$ is bounded above by $84(g-1)$, and May [7] proved that if X has nonempty boundary, this bound is $12(g-1)$.

These bounds may be considered as particular cases of the general problem of finding the minimum genus of surfaces for which a given finite group G is a group of automorphisms. The study of cyclic groups is a necessary preliminary to this, since it leads to limitations on the orders of elements within a general group. In this paper we consider the above problem for the case of cyclic groups of automorphisms of compact non-orientable Klein surfaces without boundary. The corresponding problem for compact orientable Klein surfaces without boundary was solved by Harvey [**5**].

2. Compact non-orientable Klein surfaces without boundary. By a non-Euclidean crystallographic (NEC) group, we shall mean a discrete subgroup Γ of the group of isometries G of the non-Euclidean plane, with compact quotient space, including those which reverse orientation, reflections and glide reflections. We say that Γ is a proper NEC group if it is not a Fuchsian group. We shall denote by Γ^+ the Fuchsian group $\Gamma \cap G^+$, where G^+ is the subgroup of G whose elements are the orientation-preserving isometries.

NEC groups are classified according to their signature. The signature of an NEC group Γ is either of the form

$$(*) \qquad \left(g;\ +;\ [m_1,\ldots,m_\tau];\ \left\{\left(n_{i1},\ldots,n_{is_i}\right)_{i=1,\ldots,k}\right\}\right)$$

or

$$(**) \qquad \left(g;\ -;\ [m_1,\ldots,m_\tau];\ \left\{\left(n_{i1},\ldots,n_{is_i}\right)_{i=1,\ldots,k}\right\}\right);$$