

DISINTEGRATION OF KMS-STATES AND REDUCTION OF STANDARD VON NEUMANN ALGEBRAS

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It is shown that any von Neumann algebra \mathfrak{M} with a cyclic and separating vector can be decomposed into factors in such a manner that the type of \mathfrak{M} is preserved under this decomposition.

Introduction. Since the classical work of von Neumann [9] on reduction theory appeared in 1949, several attempts have been undertaken by other authors to generalize the results of von Neumann to operator algebras which are not necessarily countably generated. We want to mention only a few more recent papers, namely those of Halpern [4, 5], Strătilă and Zsidó [18, 19], and Teleman [29, 30]. In Halpern [4] as well as in Strătilă and Zsidó [18], it was shown in a similar way that every von Neumann algebra of type I and type II can be decomposed into factors of type I and II, respectively. In Teleman [29, 30] Choquet theory was applied in order to obtain a decomposition into factors of an arbitrary von Neumann algebra.

In the present paper we shall use the theory of standard von Neumann algebras, as well as Choquet theory, in order to show that any von Neumann algebra \mathfrak{M} with a cyclic and separating vector can be decomposed into factors in such a manner that the factors which occur in the decomposition preserve the type of \mathfrak{M} . For a semifinite von Neumann algebra we shall prove a stronger result, namely there exists a disintegration of the traces which are defined on the positive cone \mathfrak{M}^+ of \mathfrak{M} .

It has already been shown by the work of Jurzak [6, 7] and Lance [8] and Sutherland [21, 22] that even in the “separable” reduction theory the Tomita-Takesaki theory can be used with considerable success to improve the earlier results in this subject. The first author who introduced the methods of Tomita-Takesaki theory into the reduction of general von Neumann algebras seems to be Halpern in his recent paper [5]. Also quite recently the present author has used the methods of Hilbert algebras in [13, 14], as well as the methods of left Hilbert algebras, in [15, 16] in order to develop a theory of direct integrals which does not depend on the countability axioms which occur in the theory of von Neumann.