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REPRESENTATIONS AND AUTOMORPHISMS OF THE IRRATIONAL ROTATION ALGEBRA

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Given an irrational number α, A_α is the unique C^*-algebra generated by two unitary operators, U and V, satisfying the twisted commutation relation $UV = \exp(2\pi i\alpha)VU$. We investigate separable representations of A_α which, when restricted to the abelian C^* algebra generated by V, are of uniform multiplicity m. These representations are classified by their multiplicity, a quasi-invariant Borel measure on the circle (w.r.t. rotation by the angle $2\pi\alpha$) and a unitary one cocycle.

Separable factor representations lie in this class, the measure being ergodic in this case. A factor representation is of uniform multiplicity m' on the C^* algebra generated by U, and if m, m' are relatively prime, the representation is irreducible. By use of an action of SL(2, Z) as *-automorphisms of A_α, that we construct, we arrive at a separating family of pure states of A_α whose corresponding irreducible representations provide explicit examples with m and m' occurring as any given pair of nonzero relatively prime numbers.

Introduction. We study representations of the irrational rotation algebras, a special class of C^*-algebras that has received a great deal of attention in recent years [**13–16**]. Our focus is primarily, though not exclusively, on factor and, in particular, irreducible, representations of algebras in this family. This class of algebras is parametrized by the irrational numbers in [0, 1]. To each irrational number α in [0, 1], we make correspond the C^*-algebra A_α generated by multiplications by continuous functions on **T**, the unit circle in the plane of complex numbers, and the unitary transformation on $L_2(\mathbf{T}, \nu)$ arising from rotation of **T** through the angle $2\pi\alpha$, where ν is (normalized) Haar measure on **T**. More specifically, let $M_f g$ be fg where $f \in C(\mathbf{T})$ and $g \in L_2(\mathbf{T}, \nu)$, and let $(Ug)(\exp(2\pi i\theta))$ be $g(\exp(2\pi i(\theta + \alpha)))$ for each θ in [0, 1]. Then A_α is the C^*-algebra generated by $\{M_f, U: f \in C(\mathbf{T})\}$.

Although we have described A_α in a particular representation, in the first instance, it can be characterized (uniquely, as it turns out) as a C^*-algebra generated by two unitary operators U and V satisfying a "twisted" commutation relation $UV = (\exp 2\pi i\alpha)VU$. In the representation of A_α we described, U is as noted and V is multiplication by z (the identity transform on **T**). There are several other ways of viewing A_α that will be useful to us. The rotation of **T** through the angle $2\pi\alpha$ is a