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STRONGLY ANALYTIC SUBSPACES AND STRONGLY DECOMPOSABLE OPERATORS

JON C. SNADER

Strongly analytic subspaces, in the sense of Lange, are studied, and a characterization of them in terms of Bishop's condition (β) is given. These results are used to obtain a characterization of strongly decomposable operators on a reflexive Banach space in terms of strongly analytic subspaces.

1. Introduction. In [**10**], Lange introduced the notion of strongly analytic subspaces, and investigated some of their properties. In this paper, we continue that investigation, and apply our results to the study of strongly decomposable operators. In §2, we study strongly analytic subspaces; here, the main result is Theorem 2.1 which characterizes strongly analytic subspaces in terms of a certain analytic condition due to Bishop. This result is then used to identify various sufficient conditions for a subspace to be strongly analytic, and to obtain certain stability results. In §3, Theorem 2.1 is combined with a result of Lange to obtain a characterization of strongly decomposable operators on a reflexive Banach space in terms of strongly analytic subspaces.

Since the leitmotif in this paper is the interplay between strongly analytic subspaces and Bishop's condition, we begin with their definition.

1.1. DEFINITION [**10**]. Let T be a bounded linear operator on a Banach space X. A T-invariant subspace Y is said to be *strongly analytic* for T if whenever f_n: $V \to X$, $n = 1, 2, \ldots$, is a sequence of X-valued analytic functions defined on an open subset V of the complex plane such that $\operatorname{dist}((\lambda - T)f_n(\lambda), Y) \to 0$ uniformly on compact subsets of V, it follows that $\operatorname{dist}(f_n(\lambda), Y) \to 0$ uniformly on compact subsets of V.

1.2. DEFINITION [**2**]. A bounded linear operator T on a Banach space X is said to possess *Bishop's condition* (β), or *condition* (β) if whenever f_n: $V \to X$, $n = 1, 2, \ldots$, is a sequence of X-valued analytic functions defined on an open set V of the complex plane such that $(\lambda - T)f_n(\lambda) \to 0$ uniformly (in norm) on compact subsets of V, it follows that $f_n(\lambda) \to 0$ uniformly (in norm) on compact subsets of V.