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GLOBAL POSITIVE SOLUTIONS OF SEMILINEAR ELLIPTIC PROBLEMS

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The existence of bounded positive solutions of semilinear elliptic boundary value problems of the type

$$(1.1)\qquad Lu = \lambda f(x, u), \quad x \in \Omega,$$

$$(1.2)\qquad u(x) = 0, \quad x \in \partial\Omega,$$

will be proved in unbounded domains $\Omega \subset R^n$, $n \geq 2$, with boundary $\partial\Omega \in C^{2+\alpha}$, $0 < \alpha < 1$, where λ is a positive constant and

$$(1.3)\qquad Lu = -\sum_{i,j=1}^{n} D_i\left[a_{ij}(x) D_j u\right] + m(x)u, \qquad x \in \Omega,$$

$D_i = \partial/\partial x_i$, $i = 1,\ldots,n$. The existence of a bounded positive solution of (1.1) in the entire space R^n is proved also by the same procedure. The regularity and additional hypotheses H1–H5 to be imposed on L and f are stated in §2. In particular, the assumption $f(x,0) = 0$ for all $x \in \Omega$ implies that the boundary value problem (1.1), (1.2) always has the trivial solution.

1. Introduction. Generally a positive solution of (1.1), (1.2) in Ω exists only if λ is sufficiently large, as might be expected from known results for bounded domains, see e.g. Rabinowitz [**23**]. In fact, under the extra hypotheses H8 and H9, we prove the Uniqueness Theorem 5.1: There exists a positive interval $(0, \lambda_*]$ such that (1.1), (1.2) has no nontrivial solution $u(x, \lambda)$ for any λ in this interval. However, under different conditions H6 and H7, Theorems 4.4 and 4.6 yield bounded positive solutions of, respectively, the boundary value problem (1.1), (1.2) and the differential equation (1.1) in all of R^n for arbitrary positive λ.

The physically important case [**9, 25**]

$$(1.4)\qquad -\Delta u + m(x)u = p(x)u^{\gamma} - q(x)u^{\beta}, \qquad x \in \Omega,$$

is included, where $1 < \gamma < \beta$ and p, q, m are nonnegative functions in Ω. Solutions $u(x)$ of (1.4) provide stationary states $e^{i\omega t}u(x)$ of the corresponding wave equation, often called the Klein-Gordon equation. In the case of constants p, q, m, the existence of positive solutions of (1.4) in the