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STABILITY FOR SEMILINEAR PARABOLIC EQUATIONS WITH NONINVERTIBLE LINEAR OPERATOR

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Suppose that

$$x'(t) + Ax(t) = f(t, x(t)), \qquad t \geq 0,$$

is a semilinear parabolic equation, e^{-At} is bounded and f satisfies the usual continuity condition. If for some $0 < \omega \leq 1$, $0 < \alpha < 1$, $\alpha\omega p > 1$, $\gamma > 1$,

$$\|t^{\omega} A e^{-At}\| \leq C, \qquad t \geq 1,$$
$$\|f(t, x)\| \leq C\left(\|A^{\alpha}x\|^{p} + (1 + t)^{-\gamma}\right), \qquad t \geq 0,$$

whenever $\|A^{\alpha}x\| + \|x\|$ is small enough, then for small initial data there exist stable global solutions. Moreover, if the space is reflexive then their limit states exist. Some theorems that are useful for obtaining the above bounds and some examples are also presented.

1. Introduction and the Main Theorem. Assume that A is a sectorial operator [2] on a (real or complex) Banach space X and that there exist $M_1 \geq 1$, $0 < \omega \leq 1$ such that

$$(1) \qquad \begin{array}{lll} \text{(i)} & \|e^{-At}\| \leq M_1 & \text{for } t \geq 0 \\ \text{(ii)} & \|Ae^{-At}\| \leq M_1 t^{-\omega} & \text{for } t \geq 1. \end{array}$$

Some theorems useful in determining ω are presented in §4, and an example is given in §5. For $\beta \geq 0$ let $X^{\beta} = D(A^{\beta})$ and $\|x\|_{\beta} = \|(A + 1)^{\beta}x\|$ for $x \in X^{\beta}$.

Assume that $0 < \alpha < 1$ and that V is an open set in X^{α}. Suppose that $f\colon [0, \infty) \times V \mapsto X$ is such that for every $t \geq 0$, $x \in V$ there exist $\varepsilon, c \in (0, \infty)$, $0 < \nu \leq 1$, for which

$$\|f(s_1, x_1) - f(s_2, x_2)\| \leq c\left(|s_1 - s_2|^{\nu} + \|x_1 - x_2\|_{\alpha}\right)$$

whenever $s_i \geq 0$, $x_i \in V$ and $|s_i - t| + \|x_i - x\|_{\alpha} < \varepsilon$ for $i = 1, 2$.

For $0 < \tau \leq \infty$ let $S(\tau)$ be the set of continuous functions $x\colon [0, \tau) \mapsto X$ which satisfy

(i) $x([0, \tau)) \subset V$ and $f(\cdot, x(\cdot)) \in C([0, \tau), X)$