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W^*-CATEGORIES

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A W^*-category is the categorical counterpart of a von Neumann algebra with an abstract definition equivalent to a concrete definition in terms of operators between Hilbert spaces. We develop the elementary theory of W^*-categories including modular theory and the comparison theory of objects (equivalence and quasiequivalence). We also characterize certain W^*-categories in terms of the W^*-category of projections in a von Neumann algebra, self-dual Hermitian modules for a von Neumann algebra or normal representations of a von Neumann algebra. This leads naturally to a discussion of the Morita equivalence of von Neumann algebras and of W^*-categories.

Introduction. A W^*-category is the natural generalization of a von Neumann algebra where, instead of taking the bounded linear mappings of a fixed Hilbert space as a model, we take the bounded linear mappings between a collection of Hilbert spaces. It is remarkable how easily most of the elementary results on von Neumann algebras generalize to W^*-categories. Consequently with little effort one can dispose of a relatively large body of results on W^*-categories. There are at present many interesting directions of current research where W^*-categories arise naturally: For example the representation theory of groupoids [**5**], the harmonic analysis of the action of non-Abelian groups on von Neumann algebras [**1**], [**9**], [**12**], [**21**], [**24**] the action of group duals on von Neumann algebras [**14**], [**22**], and non-Abelian cohomology in an operator algebraic context [**6**], [**23**], [**25**], [**28**].

We feel that a systematic presentation of the basic theory of W^*-categories is already overdue.

Naturally the idea of using bounded linear mappings between different Hilbert spaces is such an obvious one that this paper may have many published and unpublished forerunners quite unknown to the authors. Indeed one of us (J. E. R.) has been toying with the idea of writing such a paper for many years but initially felt that the time was not yet ripe for such a development. In any case the roots of this development go right back to the beginnings of the theory of operator algebras and perhaps the basic example of mappings between different Hilbert spaces are the