PACIFIC JOURNAL OF MATHEMATICS
Vol 121, No. 1, 1986

ω-ELONGATIONS AND CRAWLEY'S PROBLEM

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ω-elongations of $Z(p)$ by separable p-primary groups are studied. Assuming ($V = L$), direct sums of cyclic groups are characterized using ω-elongations. Also assuming ($V = L$) much information is obtained about ω-elongations of $Z(p)$ by groups which are not direct sums of cyclic groups. Finally it is shown that it is consistent that there is an uncountable group B with a countable basic subgroup such that there is a unique ω-elongation of $Z(p)$ of B.

In this paper all groups are p-primary Abelian groups. Suppose B is a separable group; i.e. $p^{\omega}B = 0$. Here $p^n B = \{p^n x\colon x \in B\}$ and $p^{\omega}B = \bigcap p^n B$. A group H is said to be an *ω-elongation* of A by B if $A \simeq p^{\omega}H$ and $B \simeq H/p^{\omega}H$. In this paper we will study whether or not the Σ-cyclic groups (i.e. direct sum of cyclic groups) can be characterized by their elongations of $Z(p)$. Our main theorem is:

THEOREM 2.2. *Assume ($V = L$). A group G is Σ-cyclic iff for every ω-elongation H of $Z(p)$ by G there is a homomorphism f from H to P such that $f(p^{\omega}H) \neq 0$. Here P is the Prüfer group generated by $\{x_n\colon n < \omega\}$ subject to the relations $px_0 = 0$ and $p^{n+1}x_{n+1} = x_0$. If we assume* MA + $\neg$CH *this criterion fails to characterize the Σ-cyclic groups.*

Following Megibben [**M**] we call a group B a *Crawley group*, if all elongations of $Z(p)$ by B are isomorphic (as groups). Crawley asked if all Crawley groups were Σ-cyclic. Megibben [**M**] showed, assuming MA + $\neg$Ch, there is a Crawley group which is not Σ-cyclic. Further he showed, assuming ($V = L$), any Crawley group of cardinality $\aleph_1$ must be Σ-cyclic. Megibben's proof is somewhat indirect. In particular, the proof using ($V = L$) involves valuated vector spaces. On aesthetic grounds it seems worthwhile to give a strictly group theoretic proof. In fact we get additional information on the structure of ω-elongations of $Z(p)$ by groups of cardinality $\aleph_1$ in L. The following result speaks of "rigid" systems of ω-elongations.