PACIFIC JOURNAL OF MATHEMATICS
Vol. 123, No. 2, 1986

COHOMOLOGY WITH SUPPORTS

E. SPANIER

In this paper we study cohomology theories on a space X with supports in a family of supports Φ. There is a uniqueness theorem asserting that a homomorphism between two cohomology theories on the space X with the same family of supports Φ which is an isomorphism for every $A \in \Phi$ is an isomorphism for every closed set $A \subset X$.

1. Introduction. By using cohomology with supports in a given family it is possible to pass from cohomology theories on X to cohomology theories on subsets of X with suitably related families of supports. In particular, compactly supported cohomology theories on a locally compact space X correspond to cohomology theories on the one-point compactification of X which vanish at infinity. Similarly, cohomology theories on a locally paracompact space X with relatively paracompact supports correspond to cohomology theories on the one-point paracompactification of X which vanish at infinity.

We also prove a uniqueness theorem for homomorphisms between additive cohomology theories with paracompact supports on finite dimensional space.

The remainder of the paper is divided into four sections. Section 2 contains the definition of a cohomology theory with supports in a family Φ, a uniqueness theorem for two cohomology theories with the same family of supports, and a characterization of cohomology with supports in suitable families in terms of limit properties.

Section 3 is devoted to the construction of cohomology theories on a space X with supports in a given family Φ from an ES theory on X. The definition of an ES theory on X is given and it is shown that given an ES theory on X and a family Φ of supports on X there is another ES theory on X with supports in Φ.

In Section 4 the relation between cohomology theories on X and on open subsets of X is studied. The concept of a cohomology theory on X concentrated on a subset $Y \subset X$ (i.e. which vanishes for every closed subset of X contained in $X - Y$) is introduced. The main result is a bijection between cohomology theories on X concentrated on an open set Y with supports in Φ and cohomology theories on Y with supports in a suitable family $\Phi|Y$.