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FUNCTIONAL HILBERTIAN SUMS

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A real Banach space is called a Functional Hilbertian Sum (FHS) if it is isometric to the direct sum of Hilbert spaces of dimension at least two via a one-unconditional basis. Various isometric permanence properties of Functional Hilbertian Sums are proved. Many of these results are the real analogues of (and also imply) known theorems concerning complex Banach spaces with one-unconditional (or "hyperorthogonal") bases. For example, it is proved that a space is FHS if and only if it equals the closed linear span of the ranges of its rank-two skew-Hermitian operators. The complex analogue due to Kalton and Wood is as follows: a complex Banach space has a one-unconditional basis provided it equals the closed linear span of the ranges of its rank-one skew-Hermitian operators. The isometries and skew-Hermitian operators on FHS spaces are completely determined and FHS spaces are isometrically classified. Skew-Hermitian operators on general real spaces with a one-unconditional basis are also completely determined, using FHS spaces in an essential manner. Various complementation results are established, insuring that under certain circumstances, one-complemented subspaces of spaces with one-unconditional bases are FHS spaces. One of these yields the real analogue of (and also implies) the theorem of Kalton and Wood that the family of complex Banach spaces with one-unconditional bases is closed under contractive projections. In the course of this investigation, several isometric invariants for real Banach spaces are introduced. Many of these are natural analogues of known invariants for complex spaces and include orthogonal projections, well-embedded spaces, Hilbert components and B. Lie algebras.

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